EPSG Wide-Zone TM Investigation and Recommendation r2

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r1-November 20, 2009

r2-June 2, 2010

Note: Release 2 of this presentation (r2, June 2010) combines the original recommendation to the OGP with two supplemental presentations, namely, a validation of Charles Karney’s TM Truth Points and an analysis of the two TM methods in ESRI’s ArcGIS. The first two were presented to the OGP in November 2009; the third in February 2010

Statement of the Issue

- EPSG Guidance Note 7 Part 2 provides a Transverse Mercator (coordinate operation method code 9807) algorithm based on J. P. Snyder (1987)
- Snyder's TM loses accuracy just outside of a UTM zone (at about 4 degrees from the CM)
- Better algorithms are available
- Wide-zone TM projections will be used in the petroleum industry if available
- GN7-2 should encourage a better TM standard by associating method 9807 with a wide-zone algorithm that is compatible with Snyder inside of a UTM zone
- Some background follows

Spherical => Ellipsoidal TM (1)

- Spherical TM is due to J.H. Lambert (1772)
  - Mathematically simple
- Ellipsoidal TM due to C.F. Gauss (1822)
  - Gauss used a double projection, first to a conformal sphere
  - All spherical TM properties cannot be simultaneously preserved in an ellipsoidal TM no matter how constructed. So there are many possible ellipsoidal TM variations that are conformal.
- Schreiber (circa 1880) allows the scale on the CM to vary (Gauss-Schreiber TM)
- L. Krüger (1912) preserves constant scale on the CM (Gauss-Krüger TM)
  - Uses Taylor series, hyperbolic functions and complex numbers to solve elliptical integrals
  - Suitable for wide zones, but not for the entire ellipsoid
  - Gauss-Krüger defines "Transverse Mercator" today

Spherical => Ellipsoidal TM (2)

- G. Boaga (1940), J.C.B. Redfearn (1948), P.D. Thomas (1952), Army Map Service (1973) and Snyder (1982, 1987), among others, provide different representations and truncations of Krüger's 1912 Taylor series algorithm, <u>generally with less accuracy in wide zones</u>
- E.H. Thompson (1945) developed (but didn't publish) closed, analytic (exact) TM formulas for the entire ellipsoid
  - Uses Jacobian elliptical integrals with complex numbers
  - Jacobian elliptical integrals are a parallel universe of uncommon, complex, differentiable transcendental functions that, like other transcendental functions (e.g. sines, cosines, hyperbolic sines and cosines), must be computed in some way (possibly a Taylor series)
  - Scale on the CM varies, so not compatible with GK
- L.P. Lee (1962) publishes Thompson's TM formulas (with attribution)
- Lee (1976) republishes Thompson and provides a mapping from Thompson TM into GK TM
  - The mapping uses incomplete elliptical integral of the second kind
  - Thompson-Lee is a GK equivalent that is accurate over the entire ellipsoid (with one known singularity)
  - Thompson-Lee is a different approach than Krüger

Spherical => Ellipsoidal TM (3)

- J. Dozier (NOAA, 1980) publishes an iterative, Newton's-Method algorithm based on Lee (1976) backed up by 11 double-column pages of C code, but no numerical example
- D. Wallis (JPL/NASA, 1992) publishes a description of an improved Lee-Dozier (iterative) algorithm with FORTRAN code but no mathematics
- J. Klotz (Germany, 1993) publishes a non-Taylor-series GK-equivalent algorithm adopted by the Canadian Hydrographic Service in 2009 reportedly sub-millimeter to 80 degrees (though evidence is not provided). Uses complex numbers and "Wallis integrals".
- NGA (National Geospatial-Intelligence Agency, 2009) publishes Dozier's TM algorithm in ISO/IEC 18026:2009 enhanced by C. Rollins for latitudes of origin other than the Equator, scale factors on the CM, and with scale and convergence formulas, but no code, no numerical example, sparse, difficult mathematics, and complex numbers
- Also in 2008 and 2009 an energetic TM discussion resumes on the Proj4 list server hugely informed by the thorough and thoroughly-documented investigations of Charles Karney of Sarnoff Corporation among others, notably Gerald Evenden, Oscar van Vlijmen and daan Strebe
- Some of Karney's contributions follow

Spherical => Ellipsoidal TM (4)

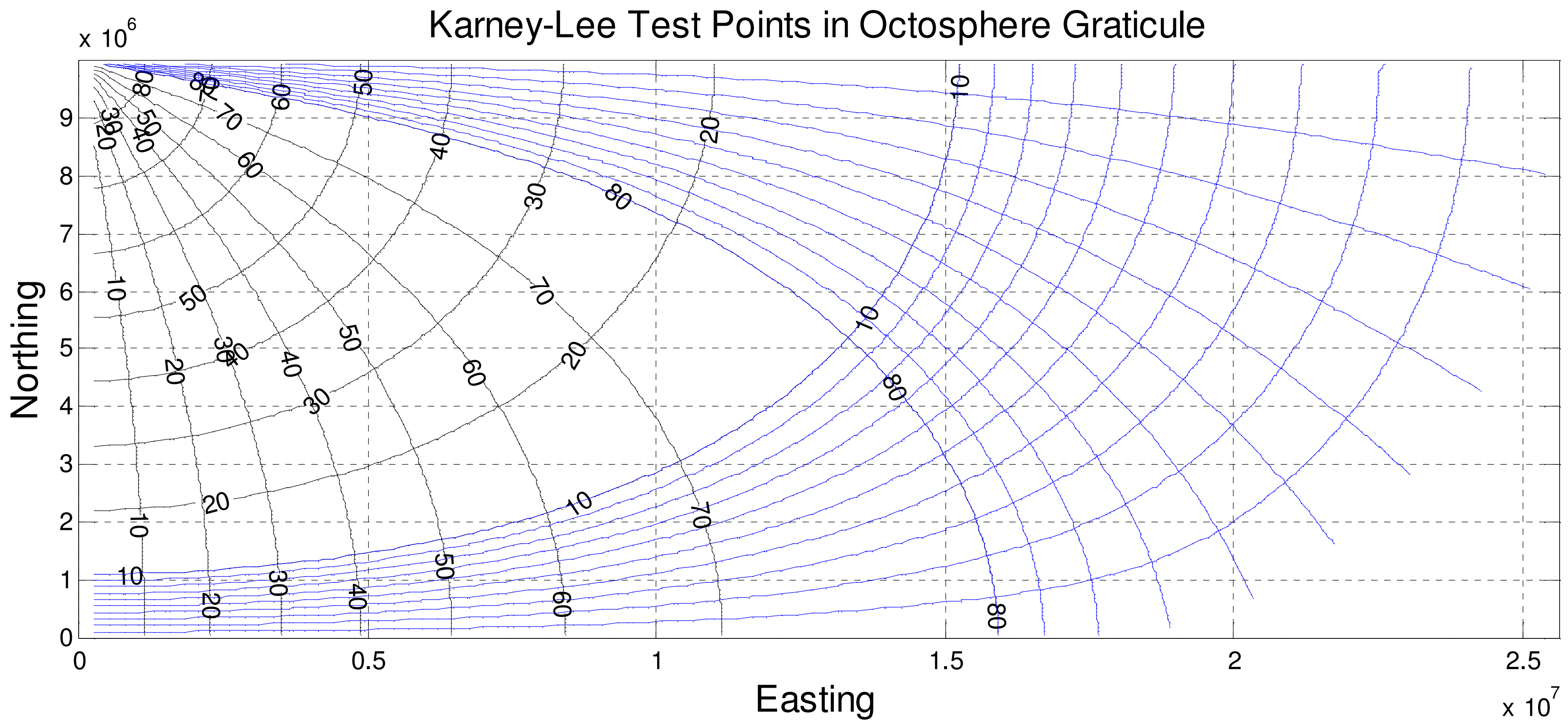
- On the Proj4 list server Karney provides:
  - Maxima code for Lee's GK TM (1976)
    - Maxima is a GNU version of Macsyma, a computer algebra system that migrated into Mathematica and Maple, presumably with native calls for the exotic math used by Lee
  - 250,000 randomly distributed truth points in the NE octosphere (0-90N, 0-90E) giving lat, lon, Northing, Easting, scale and convergence, computed using his Maxima implementation of Lee's closed GK TM. These points are used in this investigation.
  - Collections of links (some originally provided by van Vlijmen) to Scandinavian implementations of Krüger (1912), all suitable for extremely wide zones, viz. the Finnish, Swedish and Danish (due to K. Ensager & K. Poder). The Danish algorithm is marginally the best (vis-à-vis the Maxima "truth" according to Karney), has been implemented in Proj4, but (also) uses complex maths.
  - (Insanely) high order terms for an expanded Krüger, Taylor-series algorithm (if ever desired in your wildest dreams)
  - Much useful TM history with links to old, scanned documents
  - Insightful commentary (see links at the end of this presentation)

EPSG Transverse Mercator

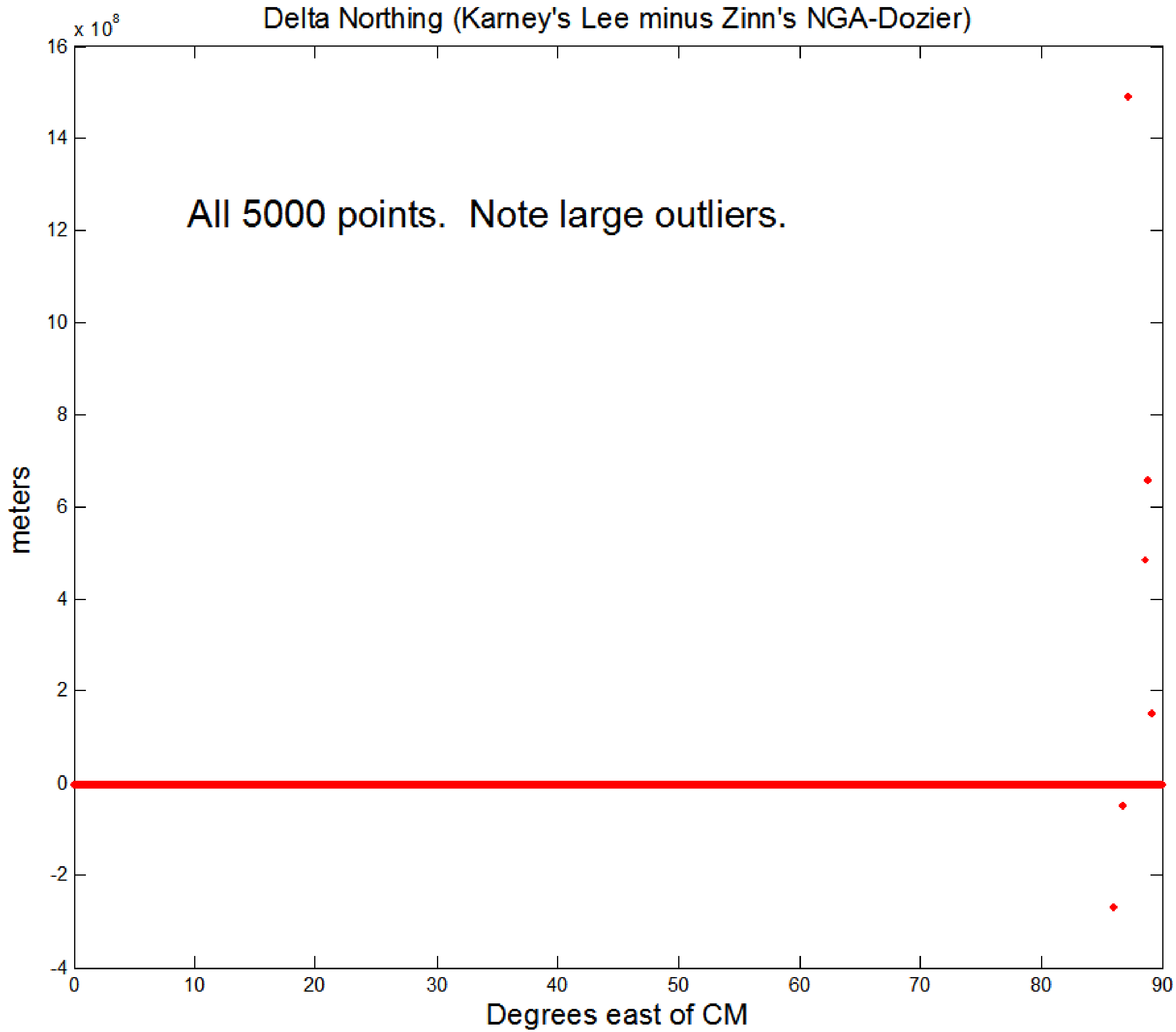
- Among the Scandinavian candidates, the Finnish algorithm:
  - Is easily available on the Internet
  - Is best documented (albeit in Finnish)
  - Is mathematically clear with hyperbolic functions but no complex numbers
  - Forward and reverse are symmetrical, elegant and brief. See code at end of this presentation.
  - Has a worked example
  - Was easy to code
  - Provides formulas for scale and convergence
  - Does not provide for a latitude of origin other than the Equator or for scale on the CM other than unity, but these enhancements are easy to implement (following Rollins' additions to Dozier for NGA)
- I recommend the Finnish algorithm as a replacement EPSG method 9807 for the reason's above and because:
  - It's in the same Gauss-Krüger, Taylor-series tradition as Snyder, i.e. none of the exotic functions of Lee and no Newton numerical methods
  - Zone width (±40°) is adequate for petroleum cartography
- Testing of the NGA-Dozier, Finnish, and EPSG-Snyder algorithms follow

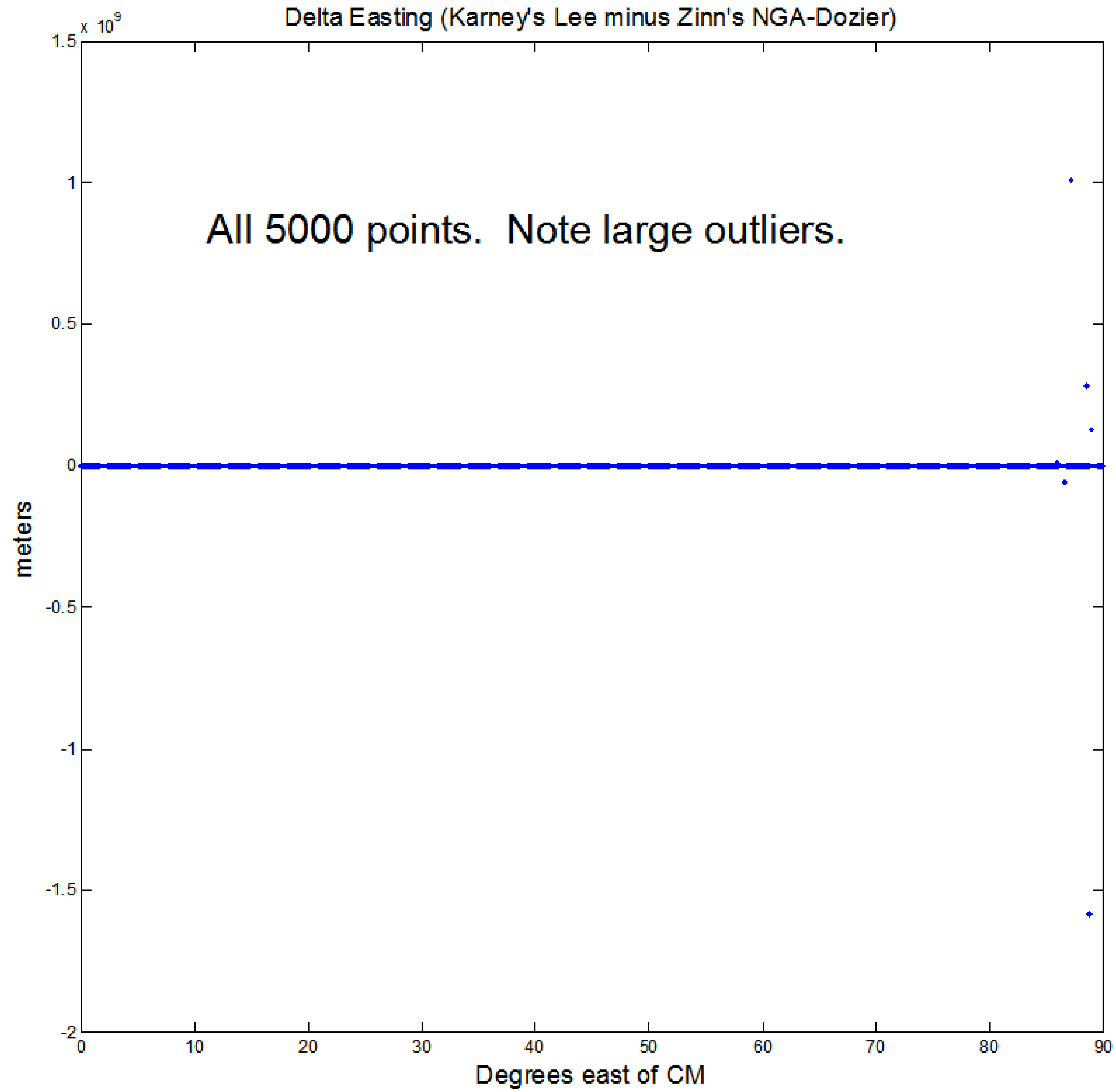
Testing EPSG TM (1)

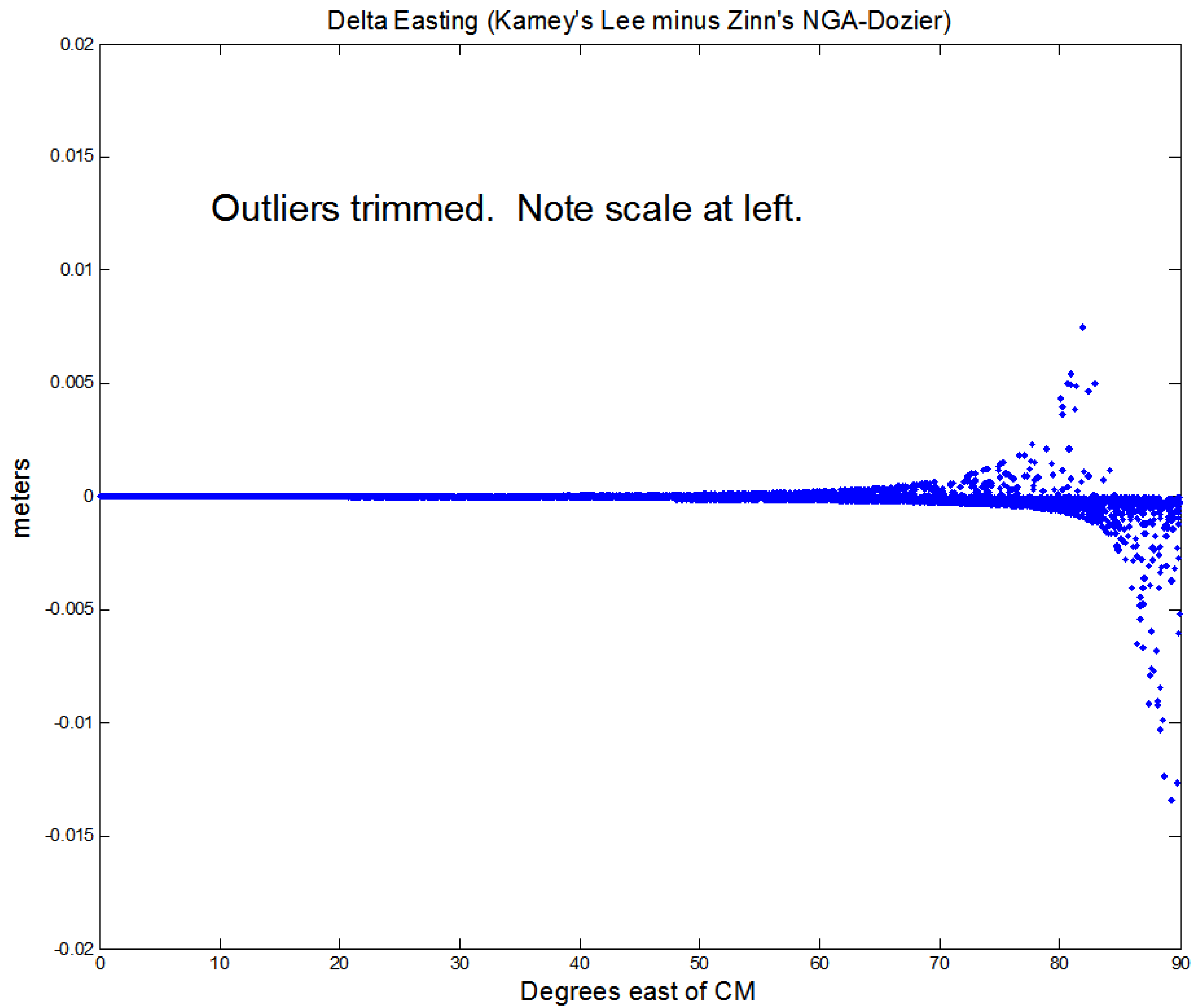
- To test the candidate algorithms far from the CM there needs to be an accepted truth answer
- The 250,000 Karney-Lee points are offered as, and can be accepted as, GK TM truth
- But to validate the truth independently I also coded the Dozier-NGA-Lee iterative Newton's-Method algorithm in Matlab using Matlab-native Jacobian elliptic integral functions and numerical quadrature in place of the incomplete elliptical integrals of the second kind (i.e. using none of Dozier's C code). Matlab natively supports complete, but not incomplete, elliptical integrals. It handles complex numbers natively. The incomplete elliptical integral Matlab code I found on the web was copyrighted and didn't seem to work anyway. Quadrature works but is slow. My NGA-Dozier-Lee implementation is only for validation and is NOT a production algorithm. ESRI have implemented Dozier to much better effect, I'm sure.
- Next Karney's test points are used to display the Lee GK graticule for the entire testing octosphere
- Then some comparisons follow

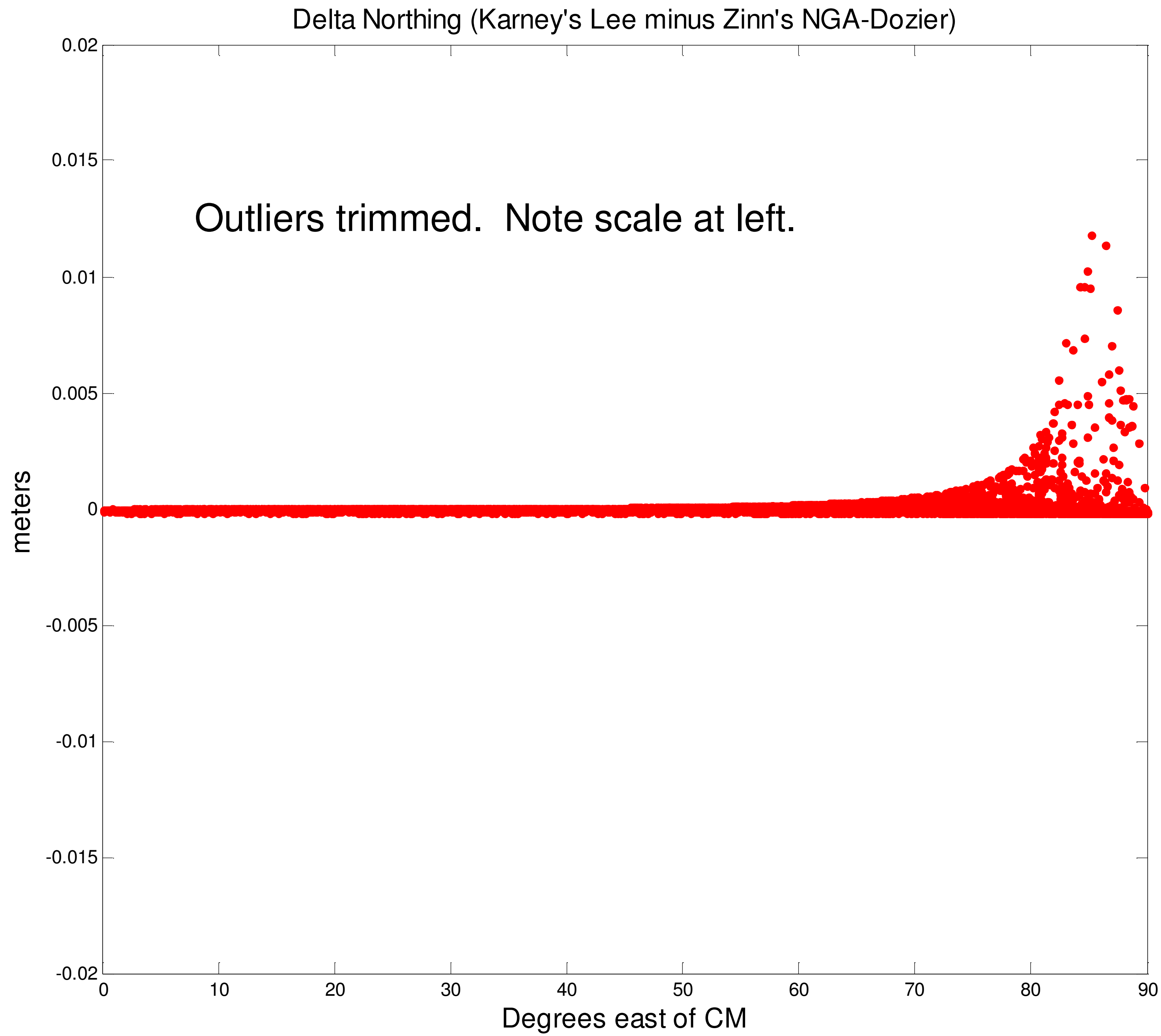


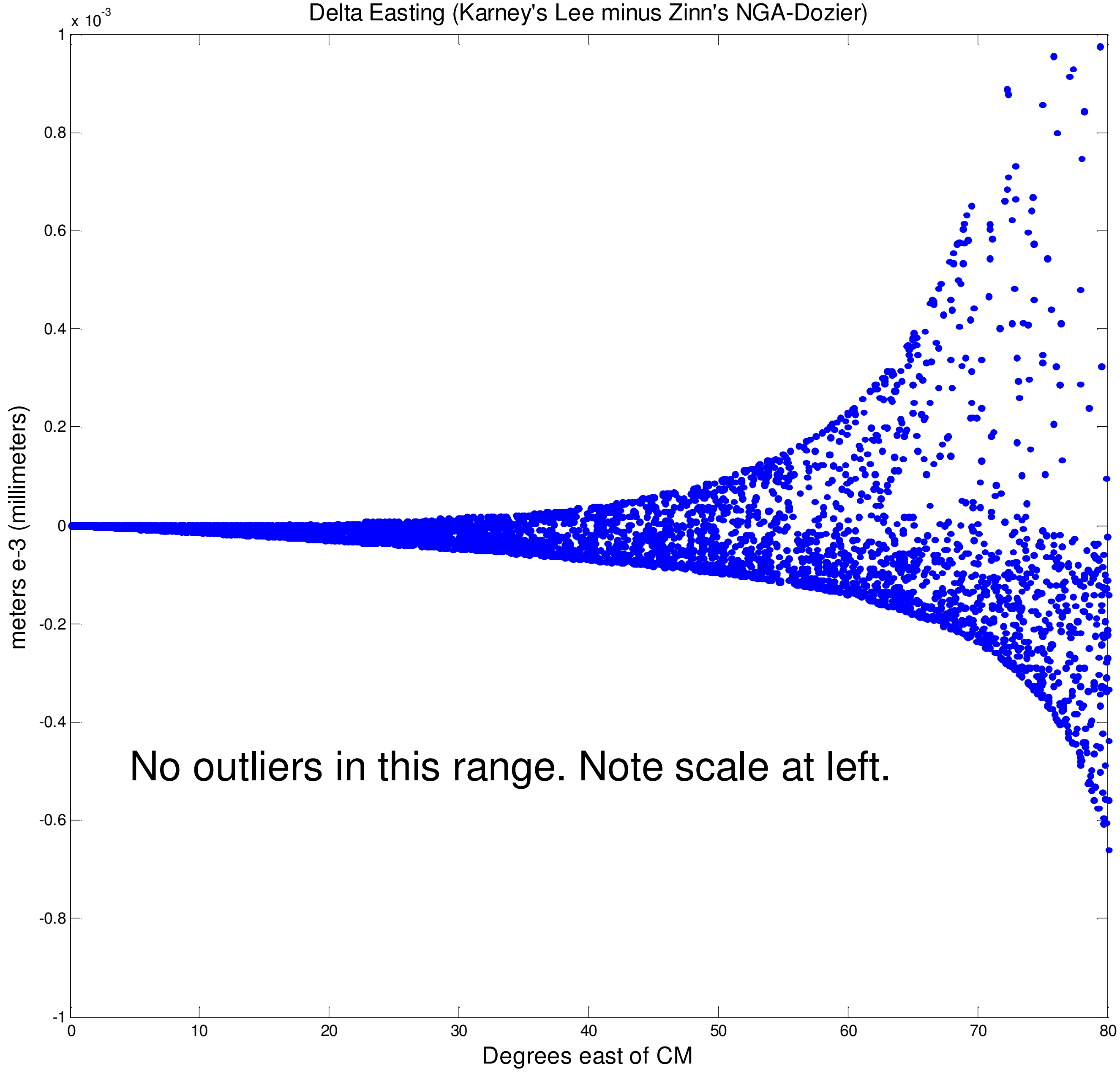
Karney-Lee Test Points in Octosphere Graticule
x 10^6
Northing
Easting
x 10^7

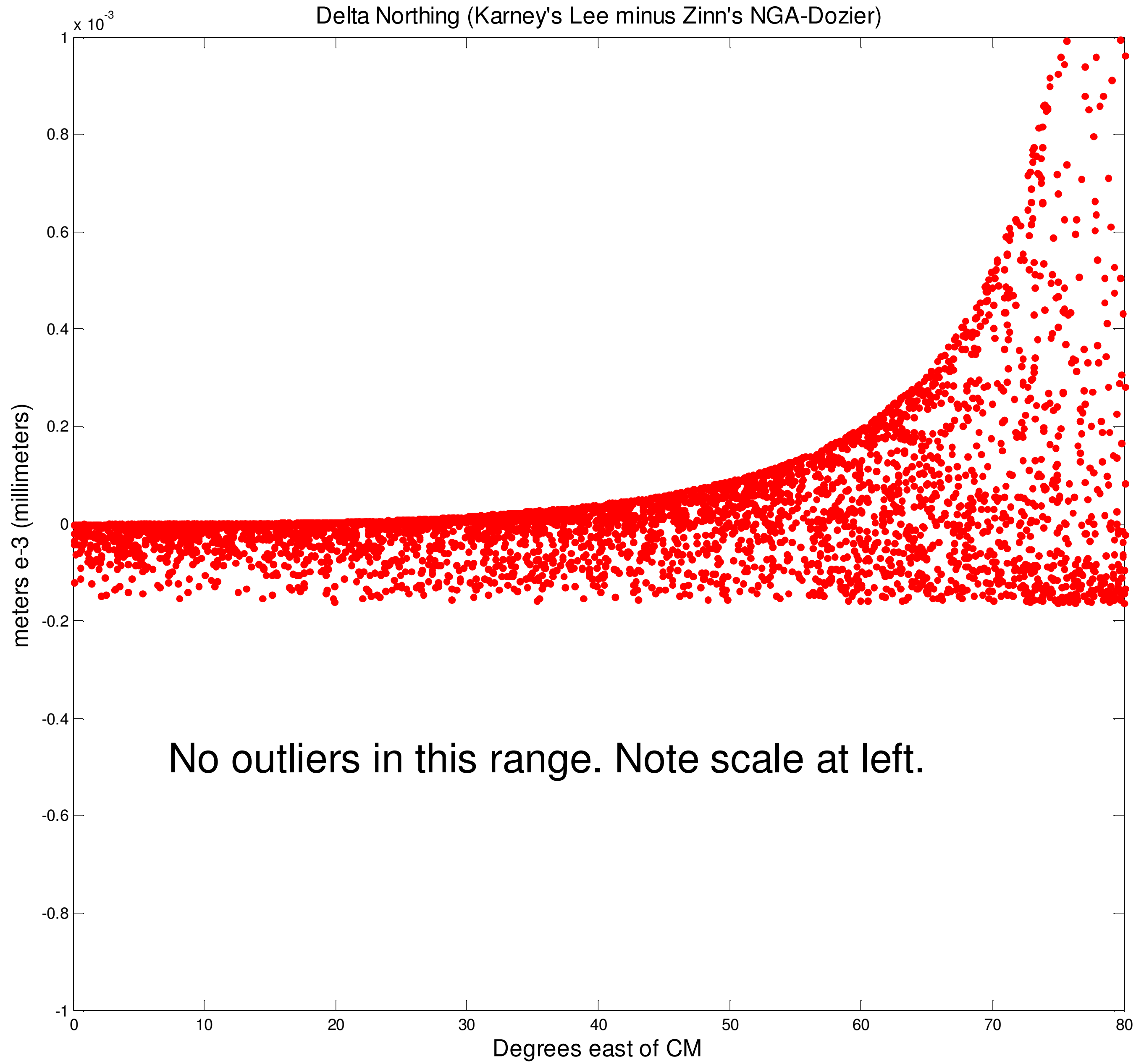

Delta Northing (Karney's Lee minus Zinn's NGA-Dozier)
x 10^8
16
14
12
10
8
6
4
2
0
-2
-4
meters
All 5000 points. Note large outliers.
0
10
20
30
40
50
60
70
80
90
Degrees east of CM


Delta Easting (Karney's Lee minus Zinn's NGA-Dozier)
x 10^9
1.5
1
0.5
0
-0.5
-1
-1.5
-2
meters
All 5000 points. Note large outliers.
0
10
20
30
40
50
60
70
80
90
Degrees east of CM


Delta Easting (Kamey's Lee minus Zinn's NGA-Dozier)
Outliers trimmed. Note scale at left.
meters
Degrees east of CM
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
0
10
20
30
40
50
60
70
80
90


Delta Northing (Karney's Lee minus Zinn's NGA-Dozier)
Outliers trimmed. Note scale at left.
meters
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02
0
10
20
30
40
50
60
70
80
90
Degrees east of CM

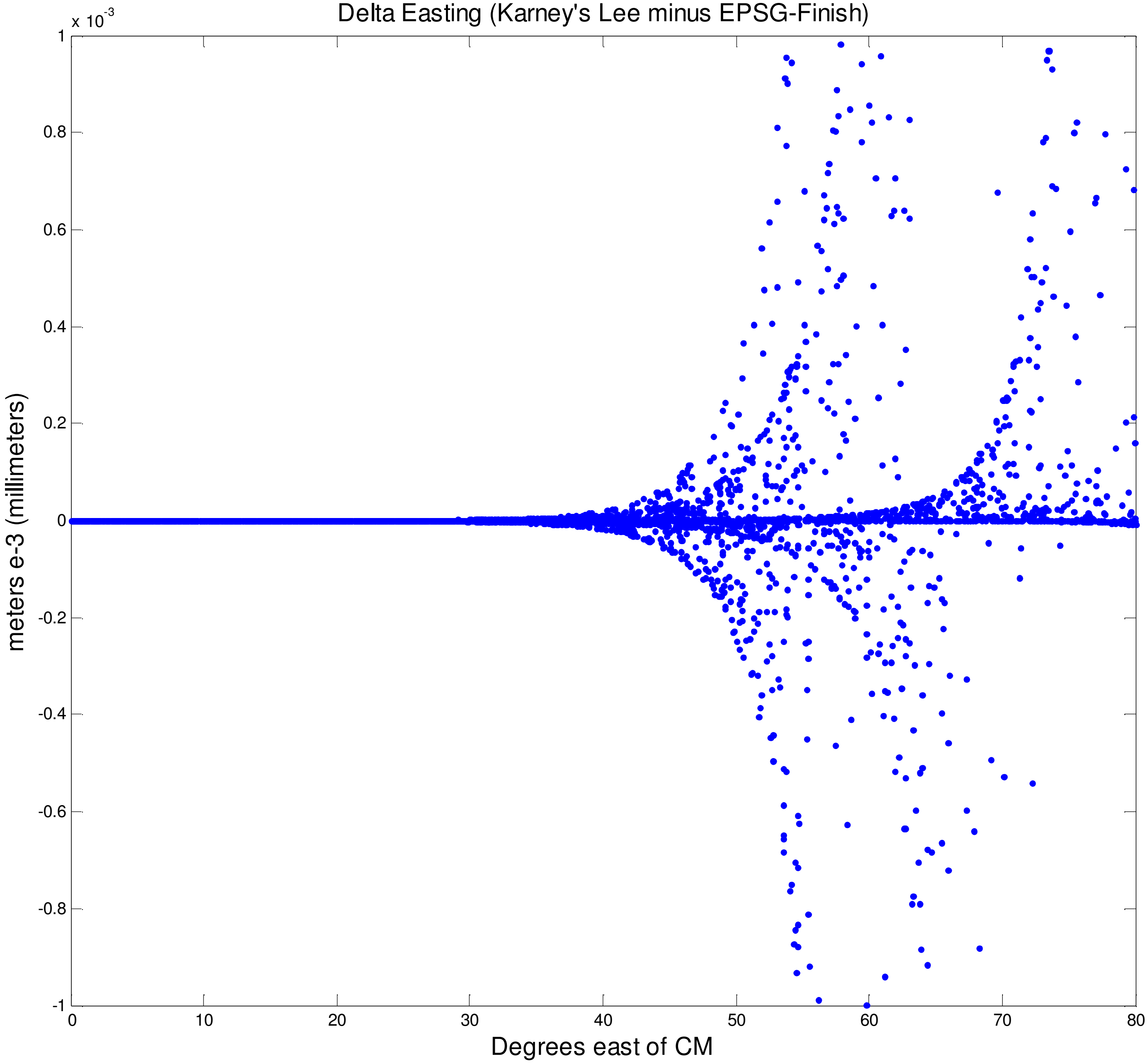

Delta Easting (Karney's Lee minus Zinn's NGA-Dozier)
x 10^-3
meters e-3 (millimeters)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
10
20
30
40
50
60
70
80
Degrees east of CM
No outliers in this range. Note scale at left.

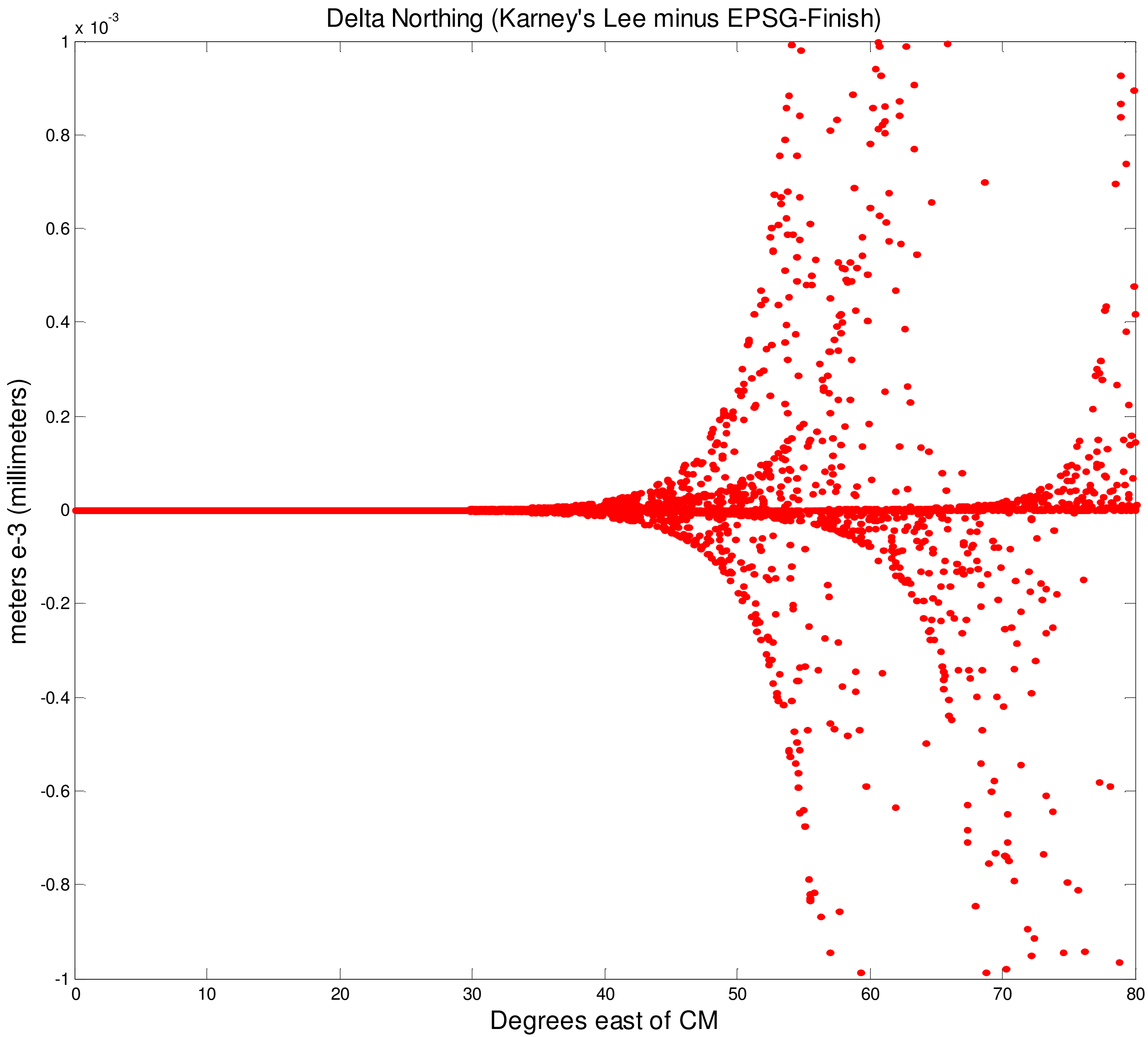


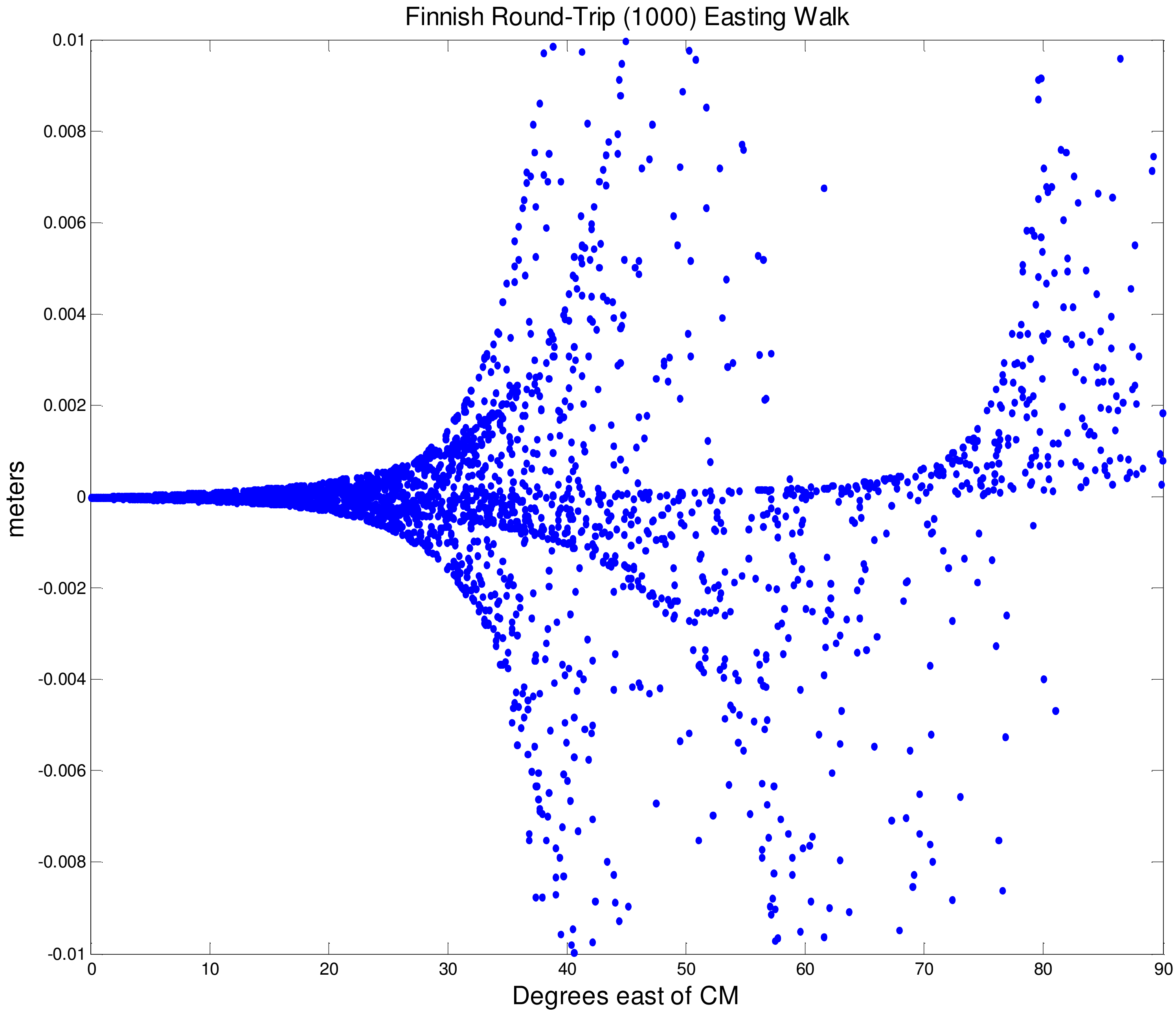
Delta Northing (Karney's Lee minus Zinn's NGA-Dozier)
x 10^-3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
meters e-3 (millimeters)
0
10
20
30
40
50
60
70
80
Degrees east of CM
No outliers in this range. Note scale at left.

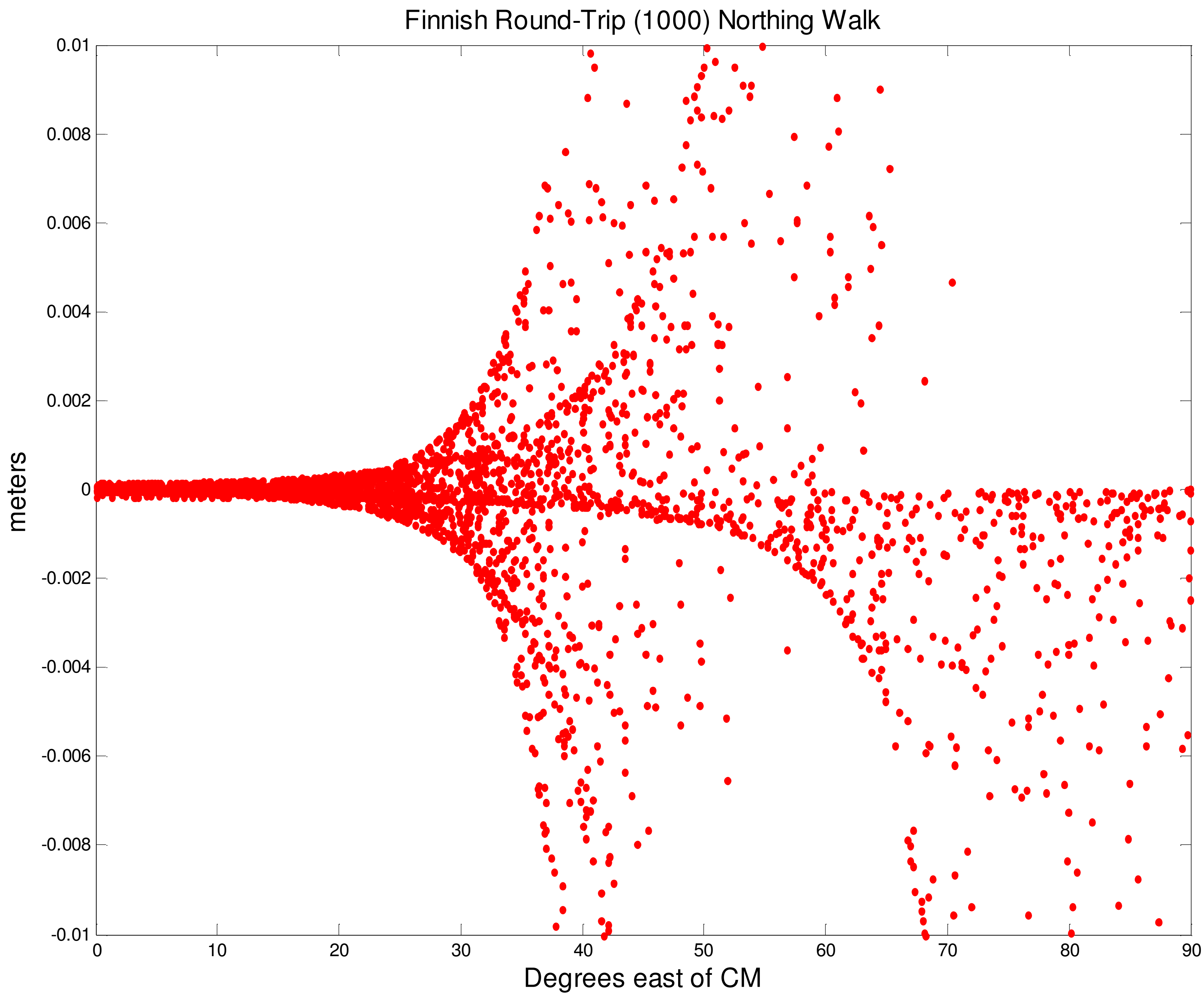
Testing EPSG TM (2)

- Only the first 5,000 of Karney's randomly distributed points are used (due to the slowness of quadrature). But the trend is clear.
- The differences between Karney's Lee in Maxima and Zinn's NGA-Dozier-Lee in Matlab are less than 0.2mm in E & N within 60 degrees of the CM and less than 1cm out to 90 degrees with the exception of fewer than 11 huge outliers (out of 5000) beyond 80 degrees
- Lee's algorithm has a known singularity on the Equator at about 82.6 degrees (at 90-e·90 exactly) that may have caused (just speculation) the NGA-Dozier outliers in this region (>80)
- Karney deprecates Dozier as unstable at some points
- NGA-Dozier-Lee is noisier in Northing than in Easting with respect to Karney's Lee, at best only 0.2mm even at 0 degrees
- Both datasets accept Lee's GK-equivalent TM as authoritative
- Given this 0.2mm@60 and 1cm@90 agreement, outliers excepted, and especially the even tighter agreement with the Finnish algorithm under 40 degrees, <u>Karney's points are accepted as the truth for further testing</u>
- The Finnish algorithm is tested next, both w.r.t. truth and round trip


Delta Easting (Karney's Lee minus EPSG-Finish)
x 10^-3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
meters e-3 (millimeters)
0
10
20
30
40
50
60
70
80
Degrees east of CM

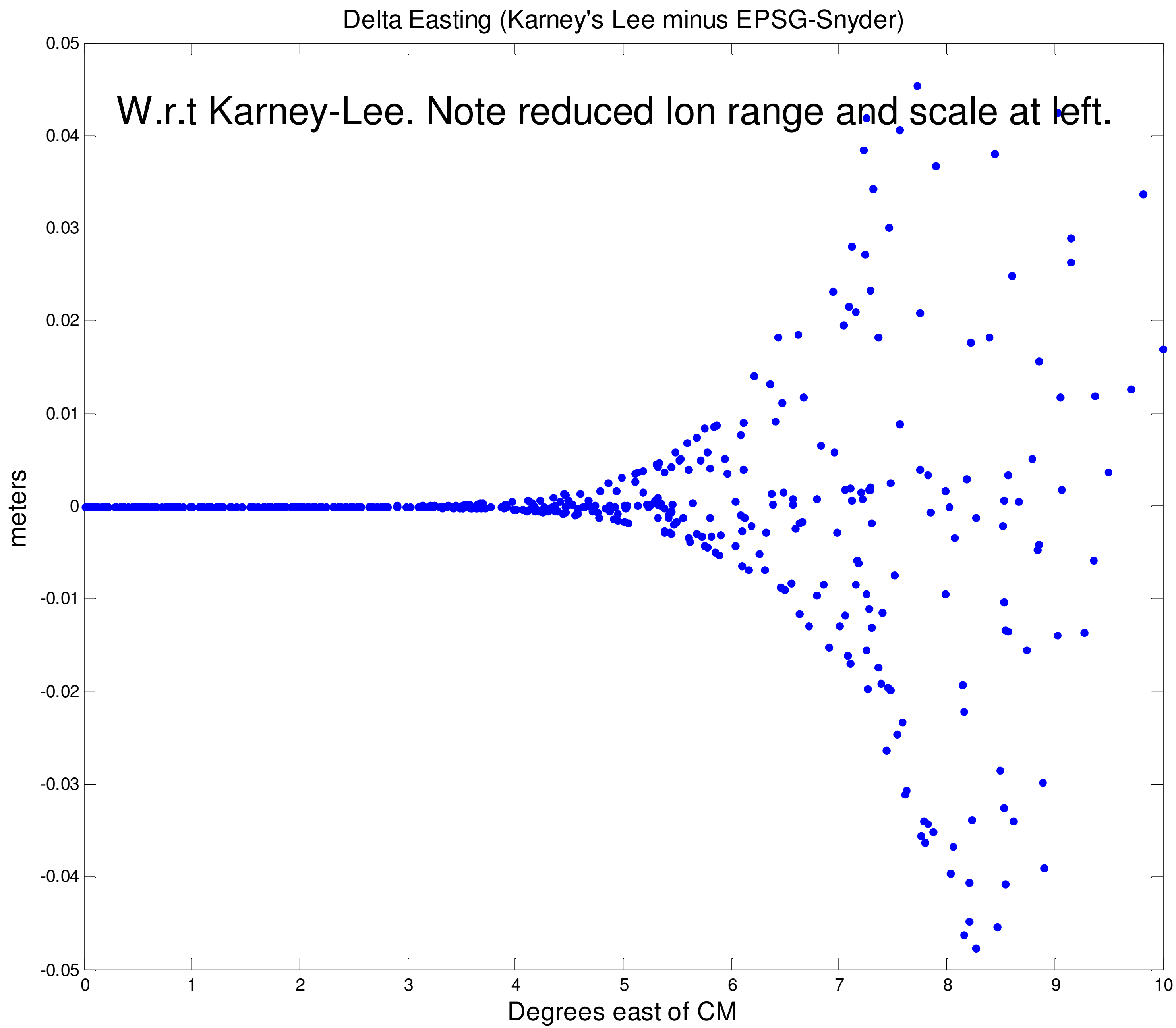

Delta Northing (Karney's Lee minus EPSG-Finish)
x 10^-3
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
meters e-3 (millimeters)
0
10
20
30
40
50
60
70
80
Degrees east of CM

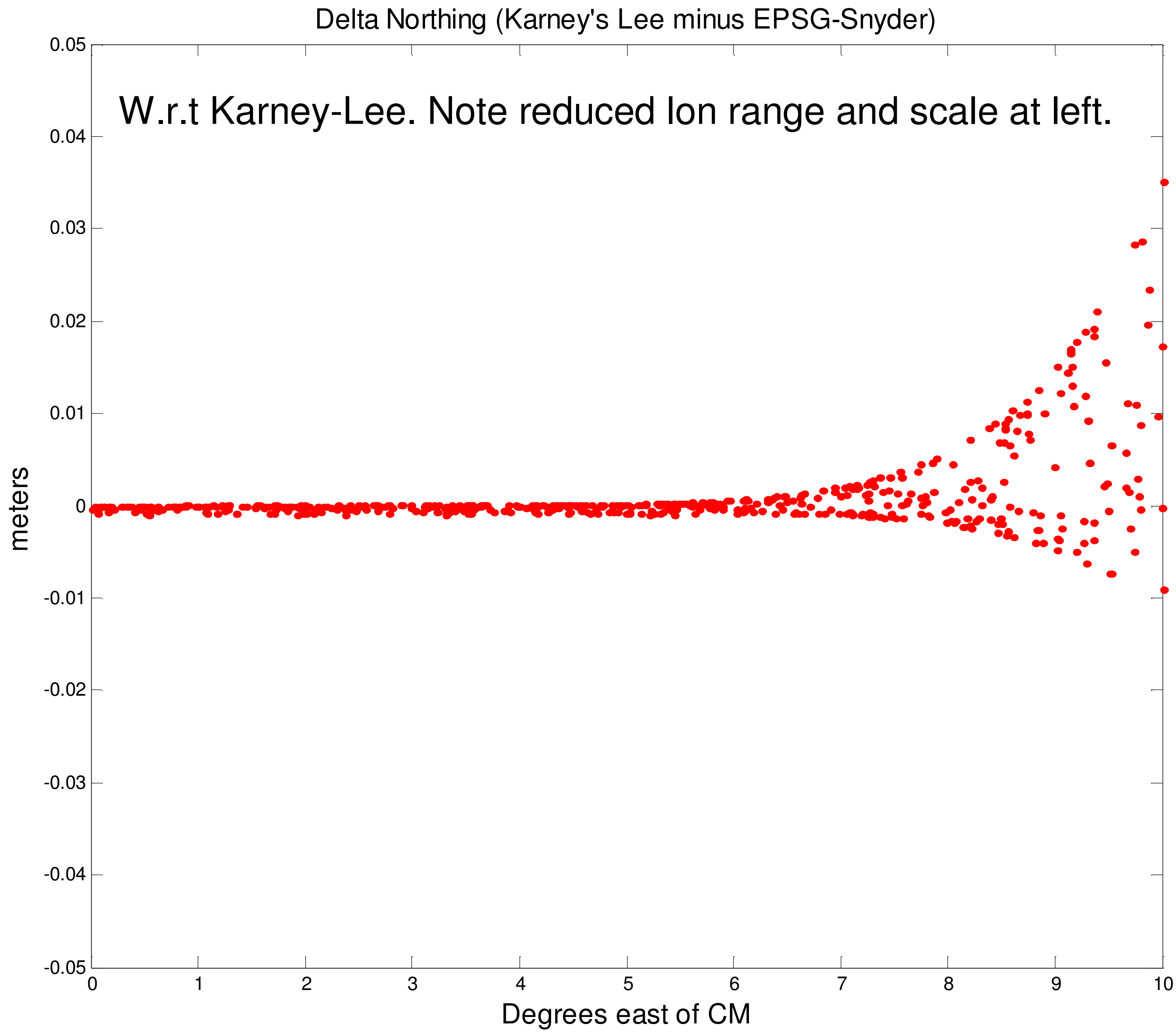

Finnish Round-Trip (1000) Easting Walk
meters
Degrees east of CM
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
0
10
20
30
40
50
60
70
80
90

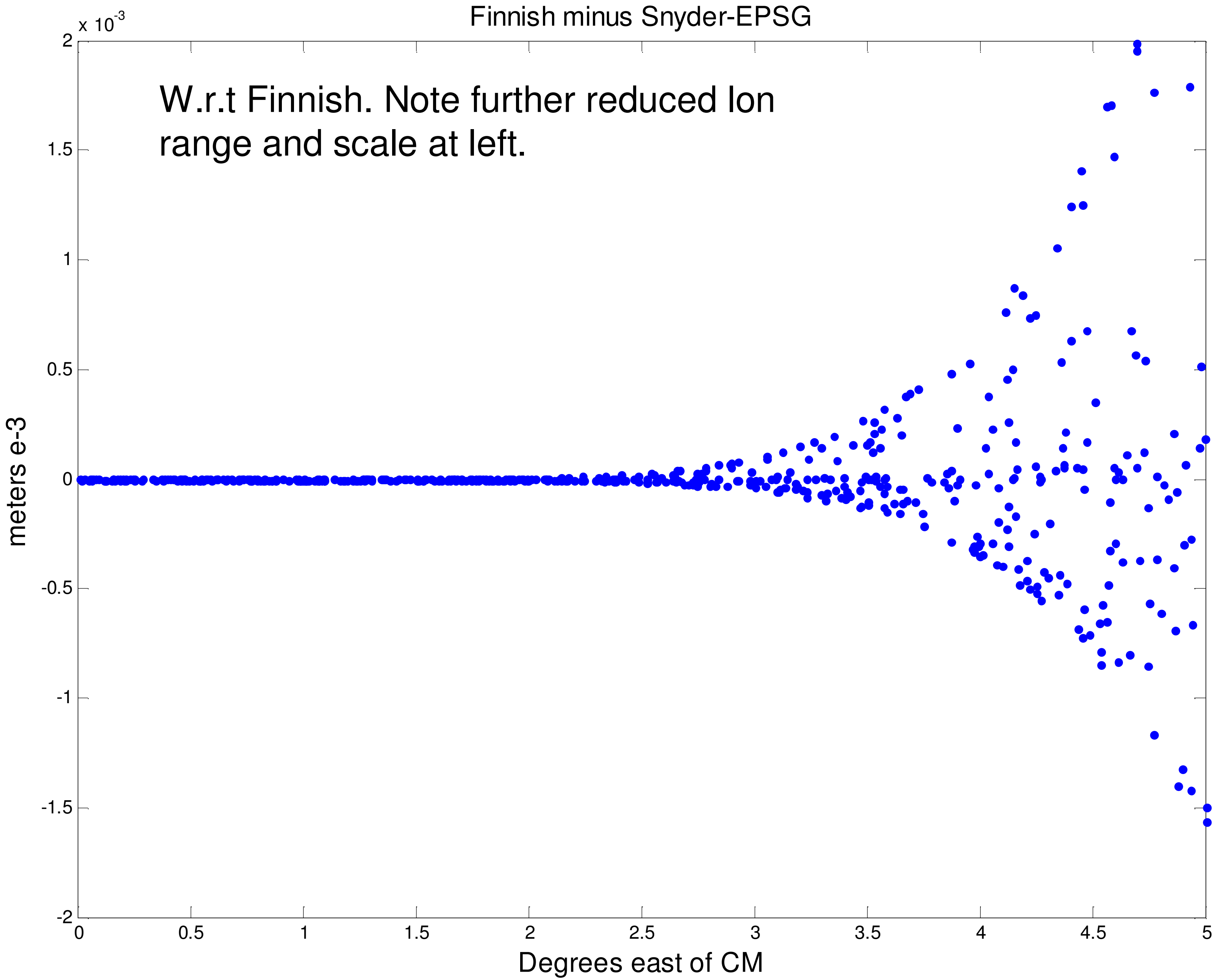

Finnish Round-Trip (1000) Northing Walk
meters
Degrees east of CM
0.01
0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
-0.01
0
10
20
30
40
50
60
70
80
90

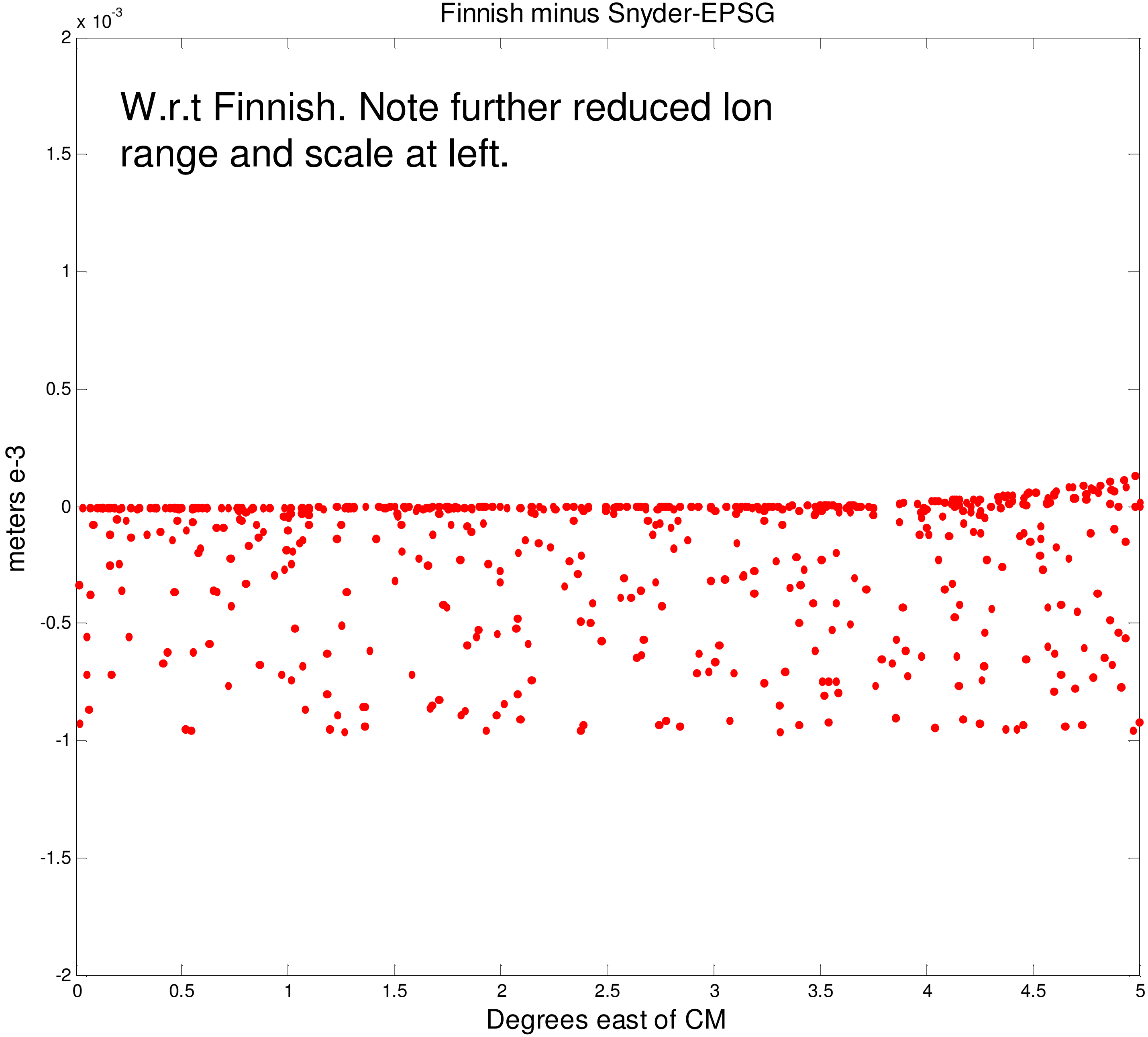
Testing EPSG TM (3)

- The Finnish algorithm is 400 times faster than Zinn’s NGA-Dozier-Lee implementation using quadrature
- The differences between the Finnish TM and the Karney-Lee truth points are less than 0.03mm in Easting & Northing out to 40 degrees from the CM (better than Zinn’s NGA-Dozier-Lee in this range), but it deteriorates thereafter
- After 1000 round trips the Finnish TM “walk” is near or well under 1cm in Easting and Northing (the APSG specification) out to 40 degrees from the CM, which is the range of acceptable agreement with our truth points
- Conclusion: The Finnish algorithm is excellent with respect to the truth and round-trip “walk” out to 40 degrees
- The current EPSG Snyder algorithm is tested next


Delta Easting (Karney's Lee minus EPSG-Snyder)
W.r.t Karney-Lee. Note reduced lon range and scale at left.
meters
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
1
2
3
4
5
6
7
8
9
10
Degrees east of CM

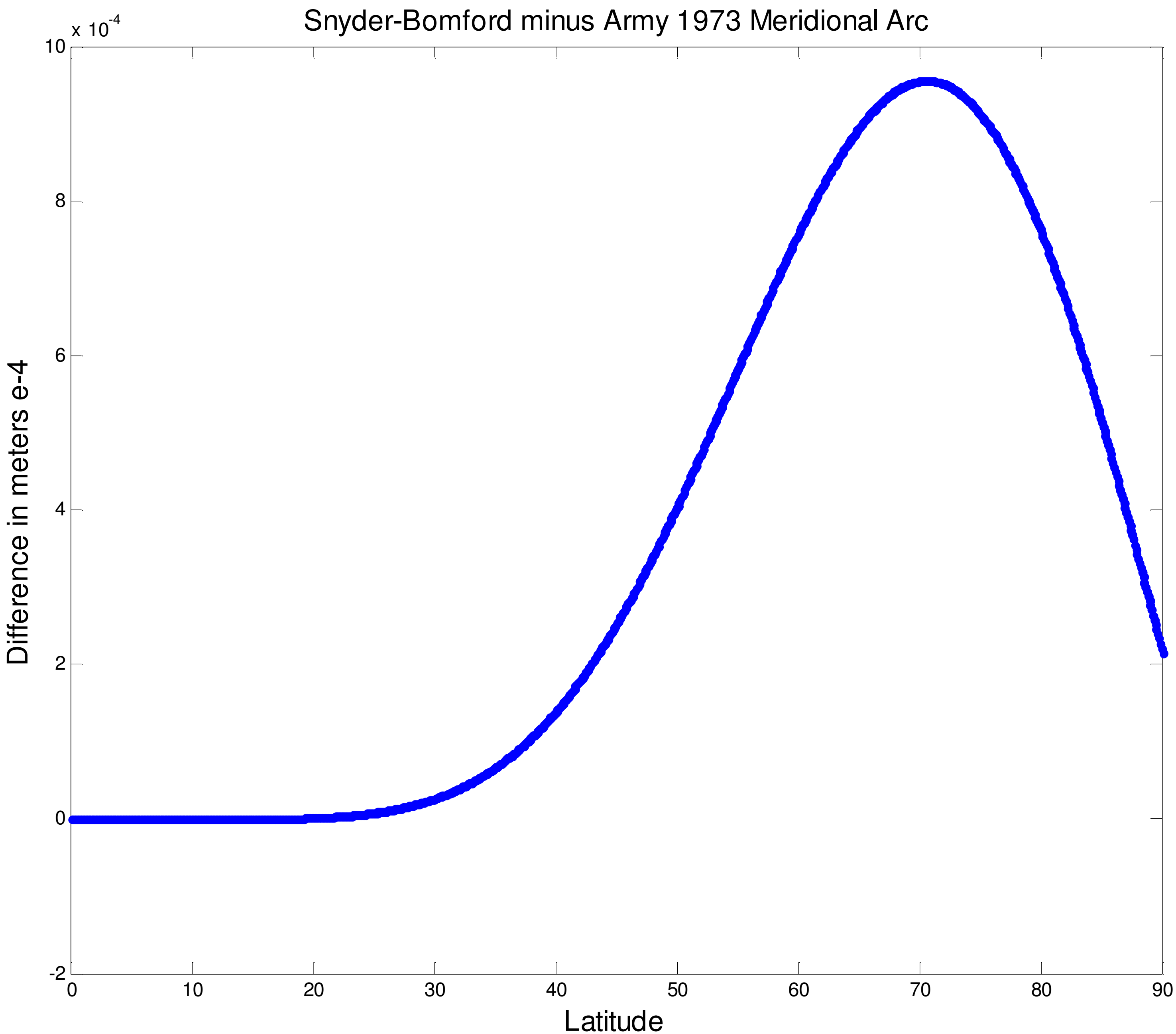

Delta Northing (Karney's Lee minus EPSG-Snyder)
W.r.t Karney-Lee. Note reduced lon range and scale at left.
meters
Degrees east of CM
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
0
1
2
3
4
5
6
7
8
9
10


Finnish minus Snyder-EPSG
x 10^-3
W.r.t Finnish. Note further reduced lon range and scale at left.
meters e-3
Degrees east of CM
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Finnish minus Snyder-EPSG
x 10^-3
W.r.t Finnish. Note further reduced lon range and scale at left.
meters e-3
Degrees east of CM
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Testing EPSG TM (4)

- Snyder is slightly faster (20%) than the Finnish algorithm
- EPSG-Snyder agrees within a millimeter of both Karney-Lee truth and the Finnish algorithm out to 4 (almost 5) degrees from the CM, but it deteriorates thereafter
- Inside of 4 degrees Snyder’s worst performance is in Northing, due wholly to the quality of the meridional arc algorithm (the same as in Bomford Appendix A), which is 1mm in error at about 70 degrees from the Equator
- The next plot differences the Snyder-Bomford and the Army 1973 arcs as a function of latitude
- Army 1973 has higher order terms than Snyder-Bomford


Snyder-Bomford minus Army 1973 Meridional Arc
x 10^-4
10
8
6
4
2
0
-2
0
10
20
30
40
50
60
70
80
90
Latitude
Difference in meters e-4

Conclusion

- Although exotic whole-ellipsoid TMs are an interesting academic entertainment, convergence and scale distortion (albeit conformal) curb their practical usefulness to narrower zones, and the mathematics are difficult
- The $\pm 40°$ zones provided by the Finnish TM are more than adequate in width for petroleum cartography
- The Finnish TM agrees with EPSG-Snyder to within 1mm from 0 to 4.5 degrees of longitude from the CM
- The exposition of the Finnish algorithm in GN7-2 will be easily understood and implemented by the lay punter
- OGP should adopt the Finnish TM for the EPSG coordinate method 9807

Appendices

- Comment on Redfearn
- Finnish TM in Matlab
- Useful Links

Redfearn (1948)

- Redfearn’s 1948 TM (an oft-cited, well-established algorithm) is about twice as good as the EPSG-Snyder TM, i.e. the spread with respect to the truth widens dramatically at 8-9 degrees rather than 4-4.5 degrees
- In the Empire Survey Review Redfearn neglects to supply an algorithm for the meridional arc
- Supplied with the Snyder-Bomford arc, Redfearn’s agreement with the Finnish would also be 1mm
- Supplied with the Army 1973 arc, Redfearn’s agreement with the Finnish within 8 degrees would be one or two orders of magnitude better than that
- In either event, Redfearn is not germane to the present discussion and further analysis is not provided

Finnish Forward in Matlab

```
function [Easting, Northing] = TMFinDir02
    (amaj, finv, lonrad0, E0, FN, latrad0, k0,
     latrad, lonrad)

flat = 1 / finv;
n = flat / (2 - flat);
A1 = amaj*(1+n^2/4+n^4/64)/(1+n);
e2 = (2 - flat) * flat;
ecc = sqrt(e2);

h1p = n/2-2*n^2/3+5*n^3/16+41*n^4/180;
h2p = 13*n^2/48-3*n^3/5+557*n^4/1440;
h3p = 61*n^3/240-103*n^4/140;
h4p = 49561*n^4/161280;

Qp = asinh(tan(latrad));
Qpp = atanh(ecc*sin(latrad));
Q = Qp-ecc*Qpp;
dlon = lonrad-lonrad0;
beta = atan(sinh(Q));

etap = atanh(cos(beta)*sin(dlon));
xip = asin(sin(beta)/sech(etap));

xi1 = h1p*sin(2*xip)*cosh(2*etap);
xi2 = h2p*sin(4*xip)*cosh(4*etap);
xi3 = h3p*sin(6*xip)*cosh(6*etap);
xi4 = h4p*sin(8*xip)*cosh(8*etap);

eta1 = h1p*cos(2*xip)*sinh(2*etap);
eta2 = h2p*cos(4*xip)*sinh(4*etap);
eta3 = h3p*cos(6*xip)*sinh(6*etap);
eta4 = h4p*cos(8*xip)*sinh(8*etap);

xi = xip+xi1+xi2+xi3+xi4;
eta = etap+eta1+eta2+eta3+eta4;

Northing = A1*xi*k0;
Easting = A1*eta*k0+E0;
```

Finnish Inverse in Matlab

```
function [latrad, lonrad] = TMFinInv01
    (amaj, finv, lonrad0, E0, FN, latrad0, k0,
     Easting, Northing)

flat = 1 / finv;
n = flat / (2 - flat);
A1 = amaj*(1+n^2/4+n^4/64)/(1+n);
e2 = (2 - flat) * flat;
ecc = sqrt(e2);

h1 = n/2-2*n^2/3+37*n^3/96-n^4/360;
h2 = n^2/48+n^3/15-437*n^4/1440;
h3 = 17*n^3/480-37*n^4/840;
h4 = 4397*n^4/161280;

xi = Northing/A1/k0;
eta = (Easting-E0)/A1/k0;

xi1p = h1*sin(2*xi)*cosh(2*eta);
xi2p = h2*sin(4*xi)*cosh(4*eta);
xi3p = h3*sin(6*xi)*cosh(6*eta);
xi4p = h4*sin(8*xi)*cosh(8*eta);

eta1p = h1*cos(2*xi)*sinh(2*eta);
eta2p = h2*cos(4*xi)*sinh(4*eta);
eta3p = h3*cos(6*xi)*sinh(6*eta);
eta4p = h4*cos(8*xi)*sinh(8*eta);

xip = xi-(xi1p+xi2p+xi3p+xi4p);
etap = eta-(eta1p+eta2p+eta3p+eta4p);

beta = asin(sech(etap)*sin(xip));
dlonrad = asin(tanh(etap)/cos(beta));

Q = asinh(tan(beta));
Qp = Q+ecc*atanh(ecc*tanh(Q));
for dex = 1:5
   Qp = Q+ecc*atanh(ecc*tanh(Qp));
end

latrad = atan(sinh(Qp));
lonrad = lonrad0+dlonrad;
```

Useful Links

- Some Proj4 list server discussion:
  - http://lists.maptools.org/pipermail/proj/2008-September/003746.html
  - http://lists.maptools.org/pipermail/proj/2008-September/003737.html
  - http://lists.maptools.org/pipermail/proj/2008-September/003811.html
  - http://lists.maptools.org/pipermail/proj/2009-February/004329.html
- NGA in ISO 18026: http://standards.iso.org/ittf/PubliclyAvailableStandards/index.html

Useful Links

- Scandinavian GK TMs:
  - [Finnish] http://docs.jhs-suositukset.fi/jhs-suositukset2/JHS154_liite1/JHS154_liite1.pdf
  - [Swedish] http://www.lantmateriet.se/upload/filer/kartor/geodesi_gps_och_detaljmatning/geodesi/Formelsamling/Gauss_Conformal_Projection.pdf
  - [Danish] http://cartography.tuwien.ac.at/ica/documents/ICC_proceedings/ICC2007/documents/doc/THEME%202/oral%201/2.1.2%20A%20HIGHLY%20ACCURATE%20WORLD%20WIDE%20ALGORITHM%20FOR%20THE%20TRANSVE.doc

Useful Links

- Karney's TM URL: http://geographiclib.sourceforge.net/html/transversemercator.html
- Dozier (NOAA): http://fiesta.bren.ucsb.edu/~dozier/Pubs/DozierUTM1980.pdf
- Klotz (German/Canadian): http://www.dfo-mpo.gc.ca/Library/337182.pdf
- Lee (New Zealand): http://utpjournals.metapress.com/content/x68715744325wm62/
- Krüger: http://bib.gfz-potsdam.de/pub/digi/krueger2.htm

TM Truth Points v3

Noel Zinn

November 21, 2009

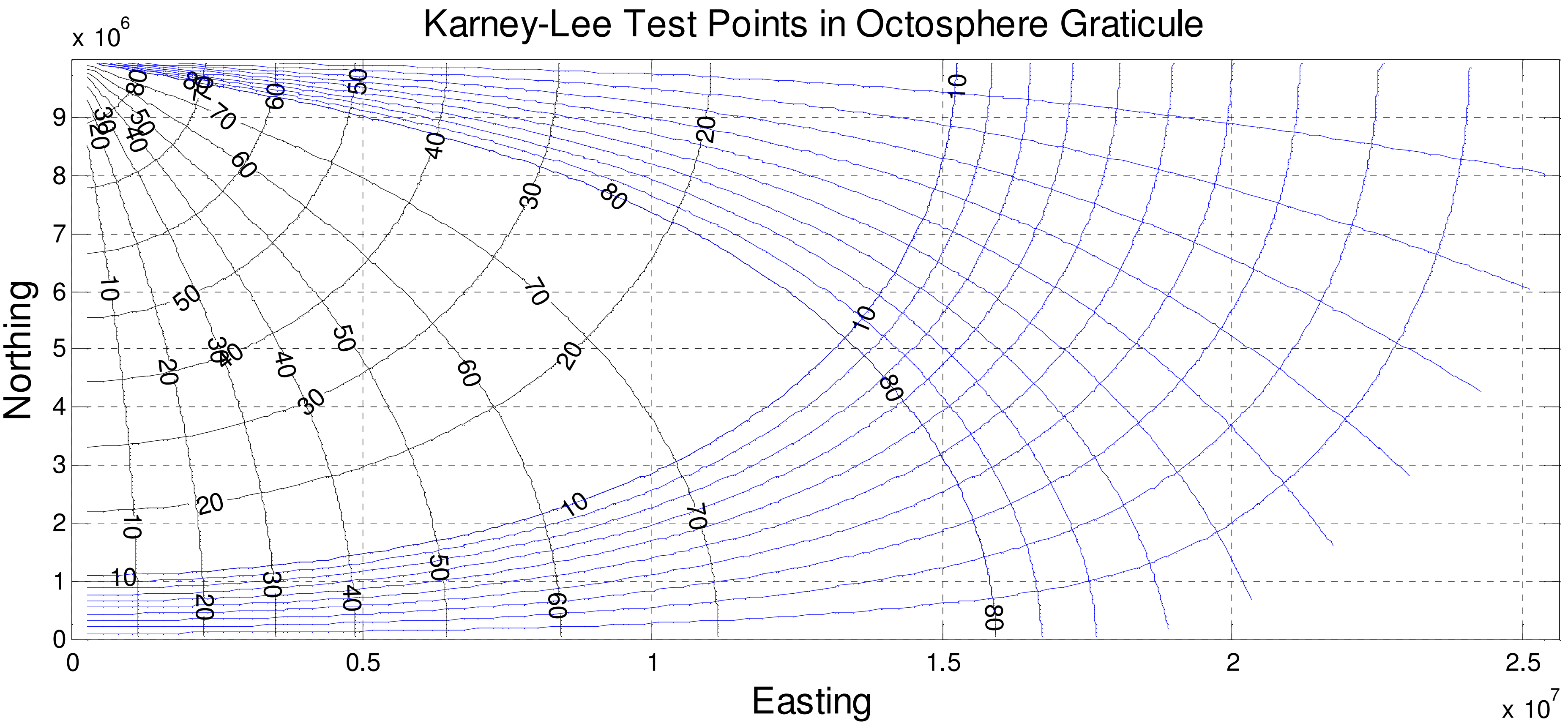
TM Truth Points

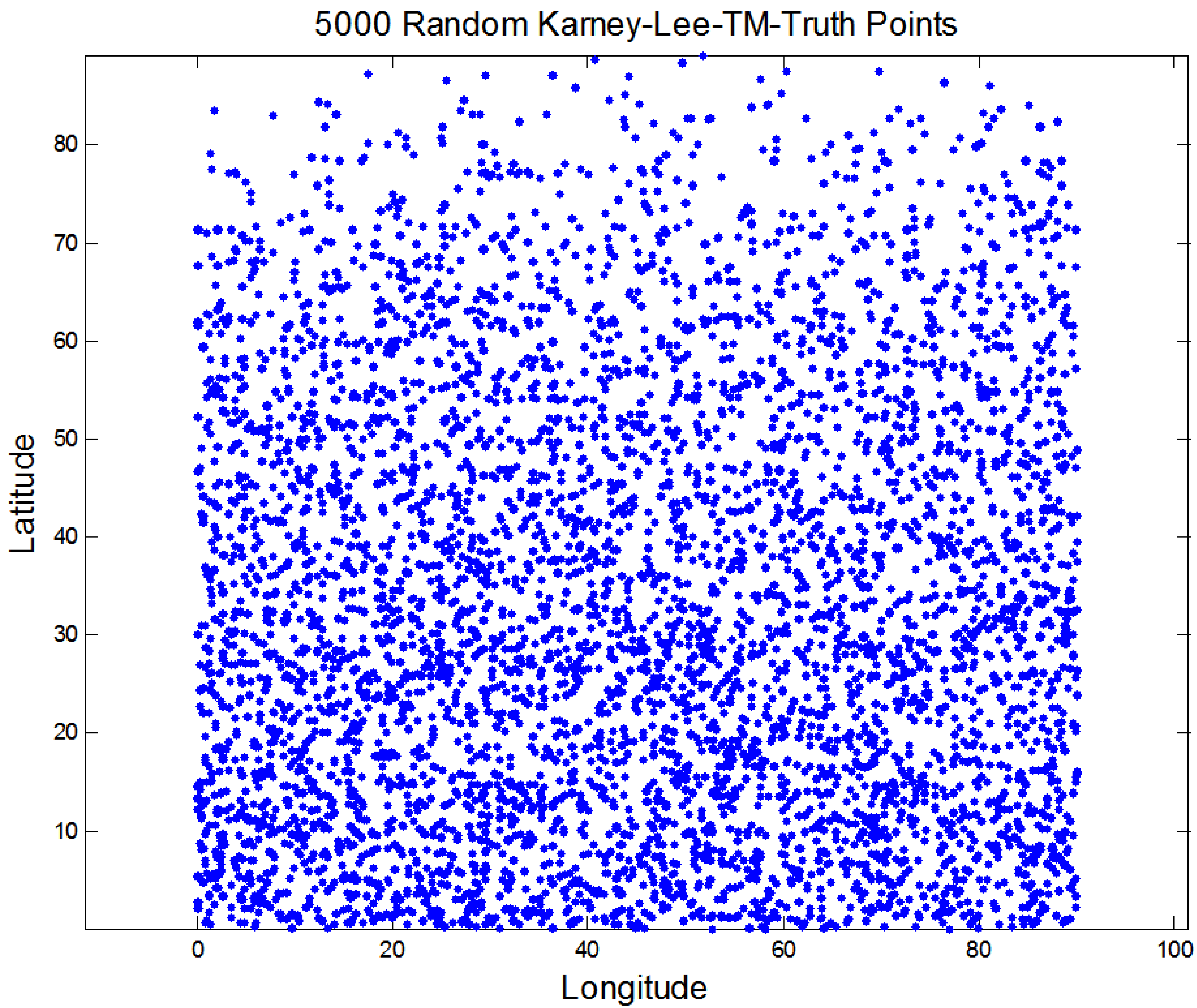
- On the Proj4 list server Charles Karney of Sarnoff Corporation provides 250,000 randomly distributed truth points in the NE octosphere (0-90N, 0-90E, plus some additional points outside this octosphere) computed using his Maxima implementation of Lee's closed GK TM
- I have satisfactorily verified these points with my Matlab implementation of Dozier's Newton iterative version of Lee
- The first 5000 of these points are used in the analysis supporting my TM recommendation to OGP
- The following slides exhibit the distribution of these 5000 points
- Any 5000-point sample will exhibit the same distribution
- In this sense the 250,000 points are randomly distributed

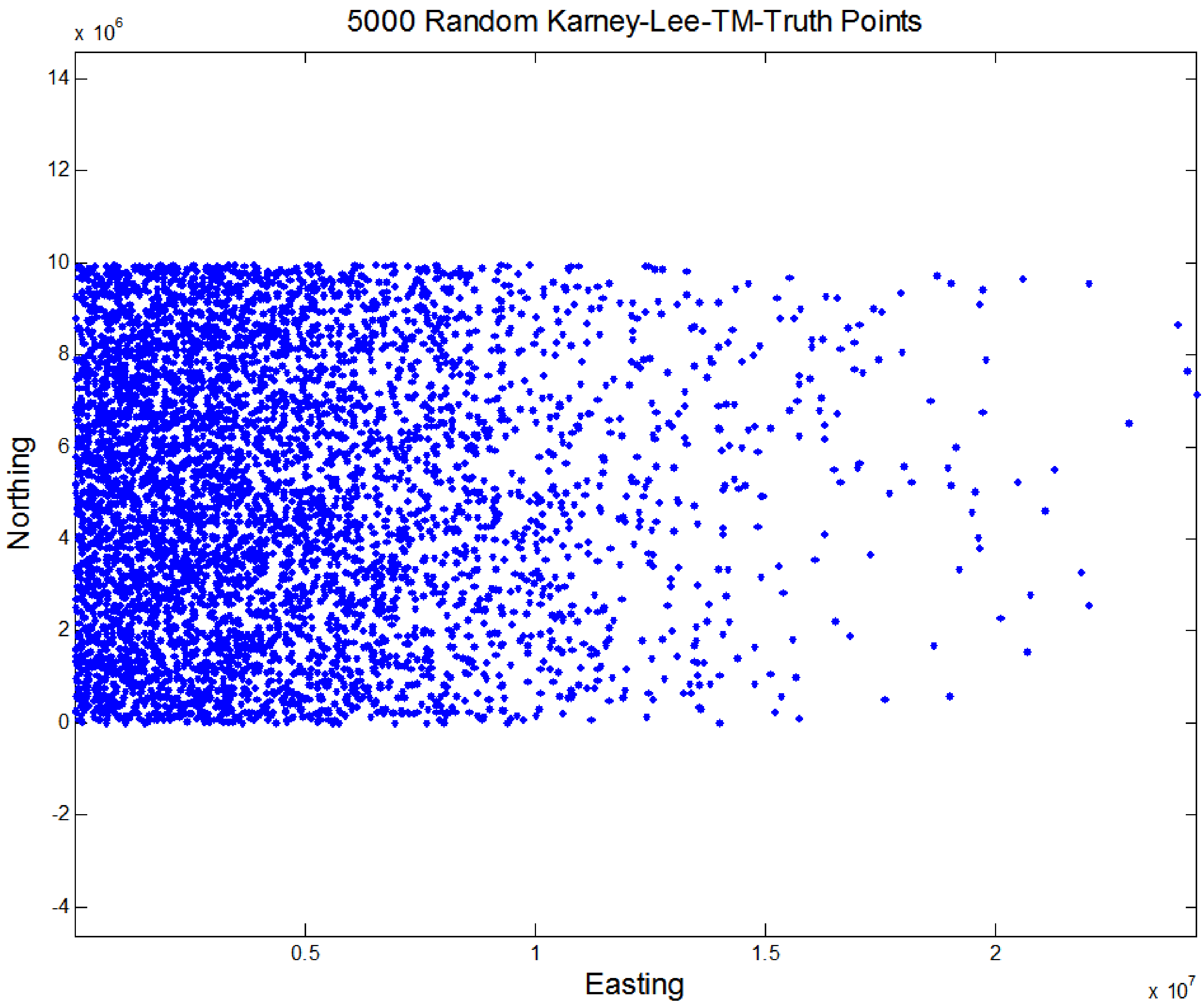
Description of Following Plots

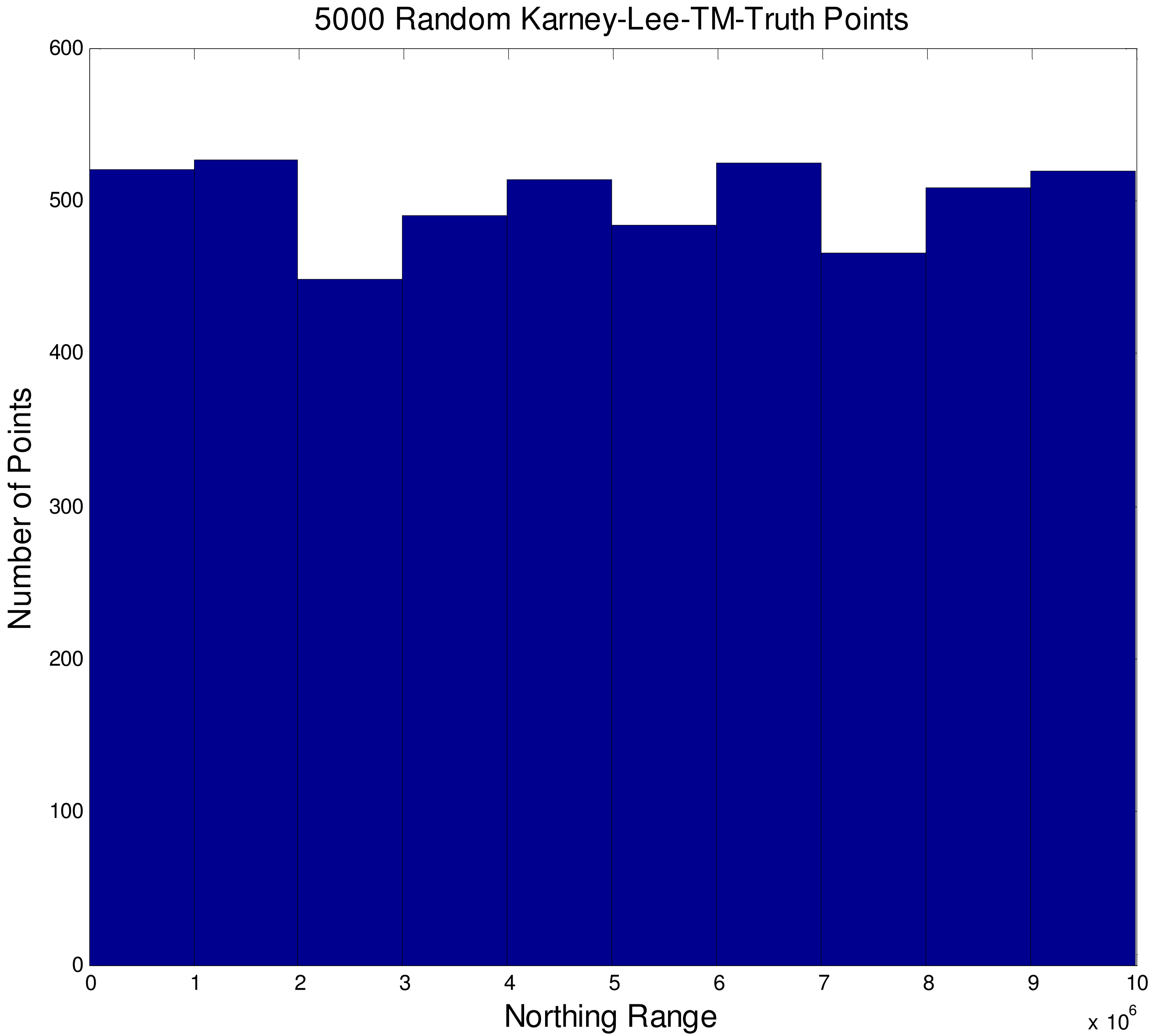
- First the octosphere graticule is reproduced in TM space
- Then the 5000 points are plotted in lat/lon and E/N space
- Then histograms of the distributions in Northing, Easting, Latitude and Longitude are offered
- Finally, the 5000 points are plotted in topocentric (ECEF) coordinates in plan view, which is the orthographic projection
- The final plot shows a uniform distribution of points in ECEF space

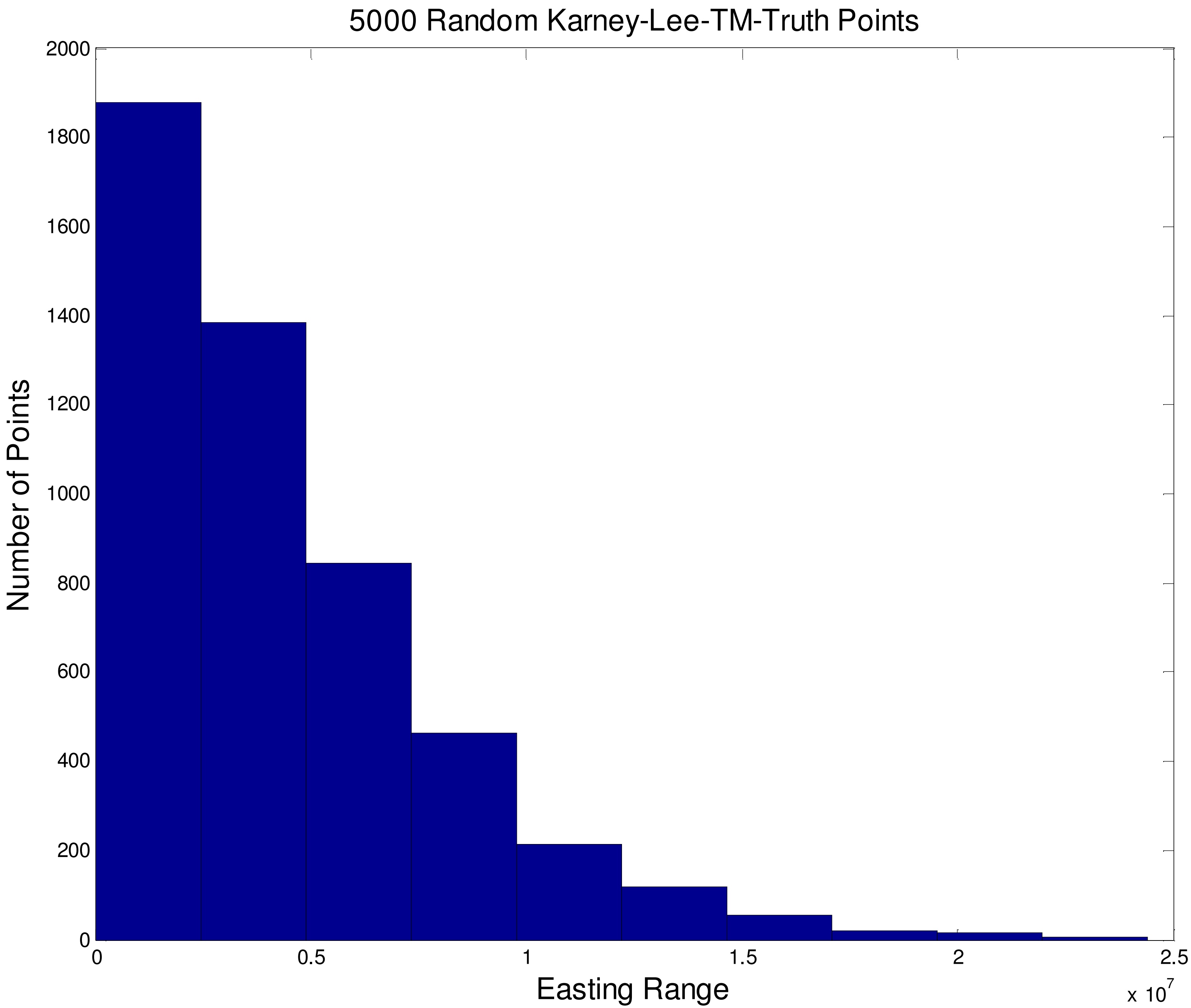
Graticule Plotted in TM 0-90N, 0-90E

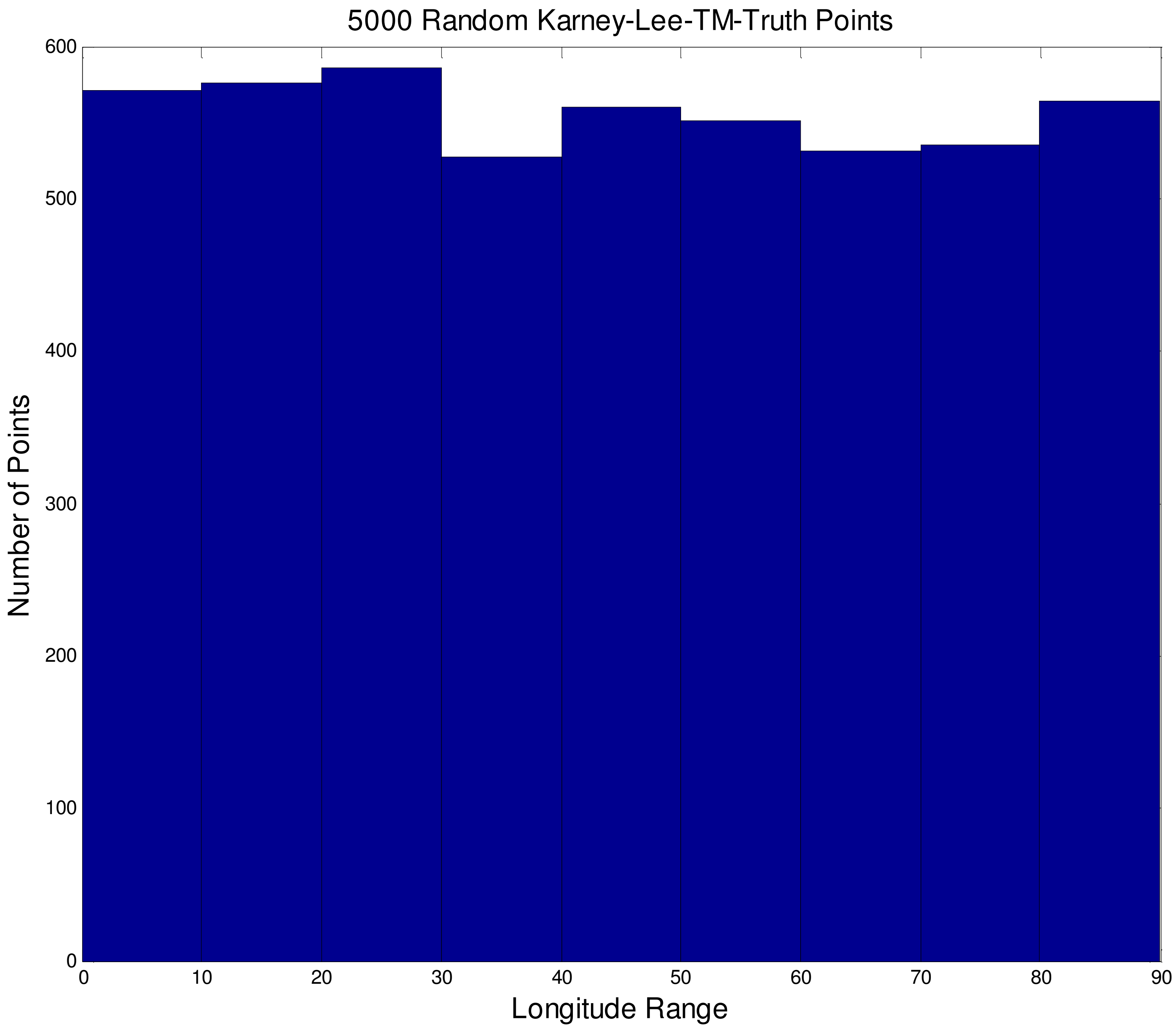


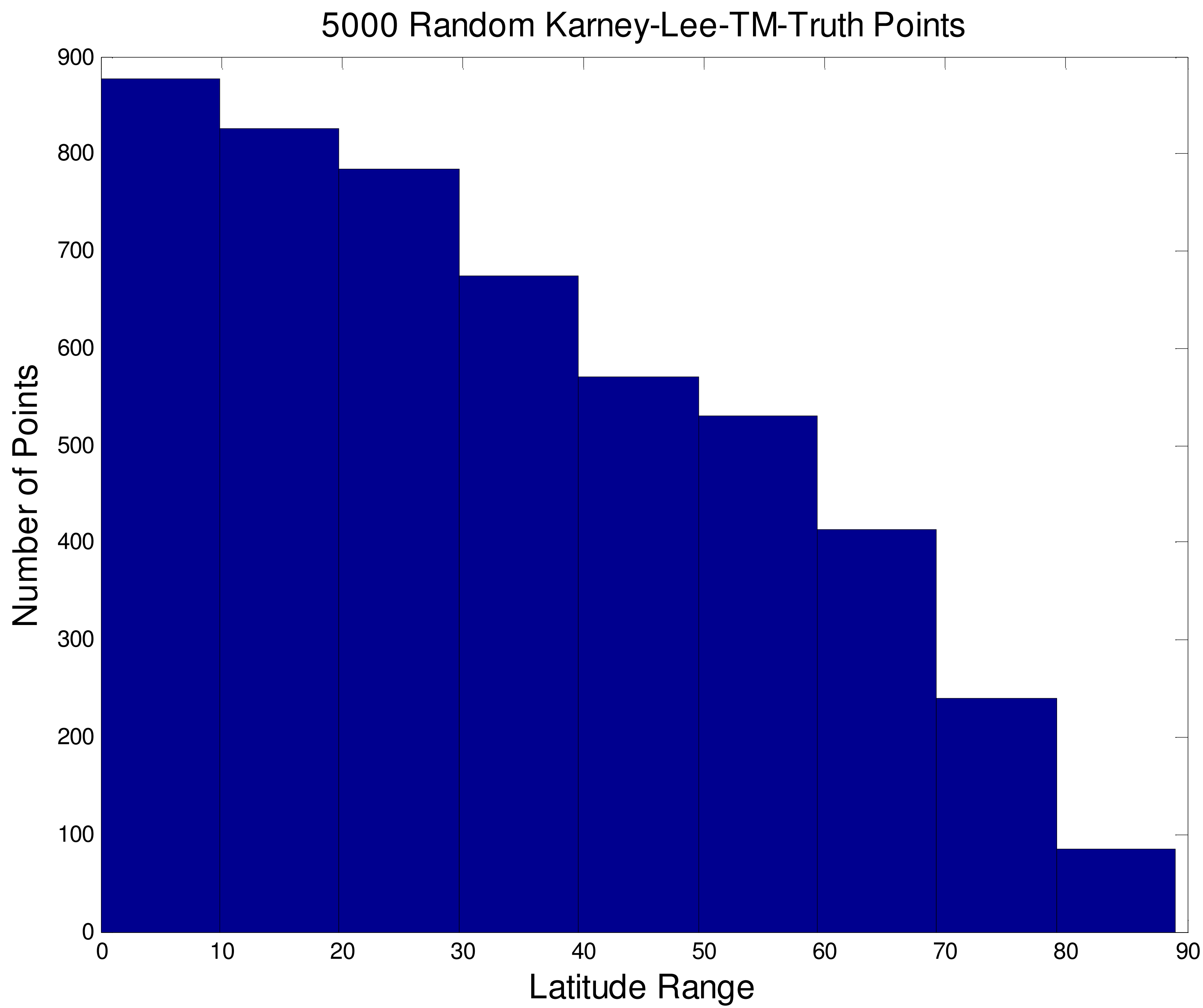

5000 Random Karney-Lee-TM-Truth Points
Latitude
Longitude
0
20
40
60
80
100
10
20
30
40
50
60
70
80

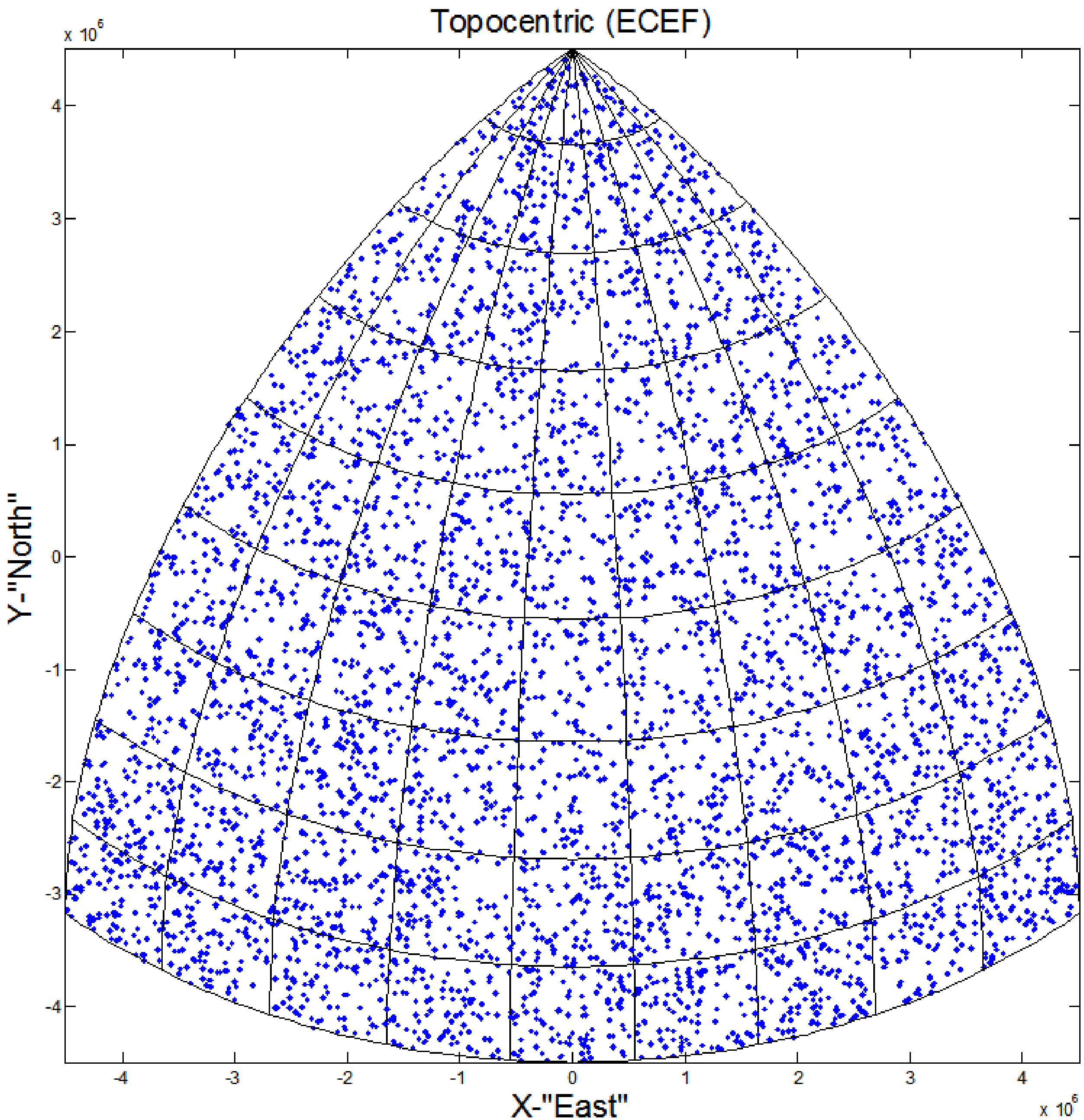

5000 Random Karney-Lee-TM-Truth Points
x 10^6
14
12
10
8
6
4
2
0
-2
-4
Northing
0.5
1
1.5
2
Easting
x 10^7


5000 Random Karney-Lee-TM-Truth Points
Number of Points
Northing Range
x 10^6
0
100
200
300
400
500
600
0
1
2
3
4
5
6
7
8
9
10


5000 Random Karney-Lee-TM-Truth Points
Number of Points
Easting Range
x 10^7
0
200
400
600
800
1000
1200
1400
1600
1800
2000
0
0.5
1
1.5
2
2.5


5000 Random Karney-Lee-TM-Truth Points
Number of Points
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80
90
Longitude Range


5000 Random Karney-Lee-TM-Truth Points
Number of Points
0
100
200
300
400
500
600
700
800
900
0
10
20
30
40
50
60
70
80
90
Latitude Range


Topocentric (ECEF)
x 10^6
4
3
2
1
0
-1
-2
-3
-4
Y-"North"
-4
-3
-2
-1
0
1
2
3
4
X-"East"
x 10^6

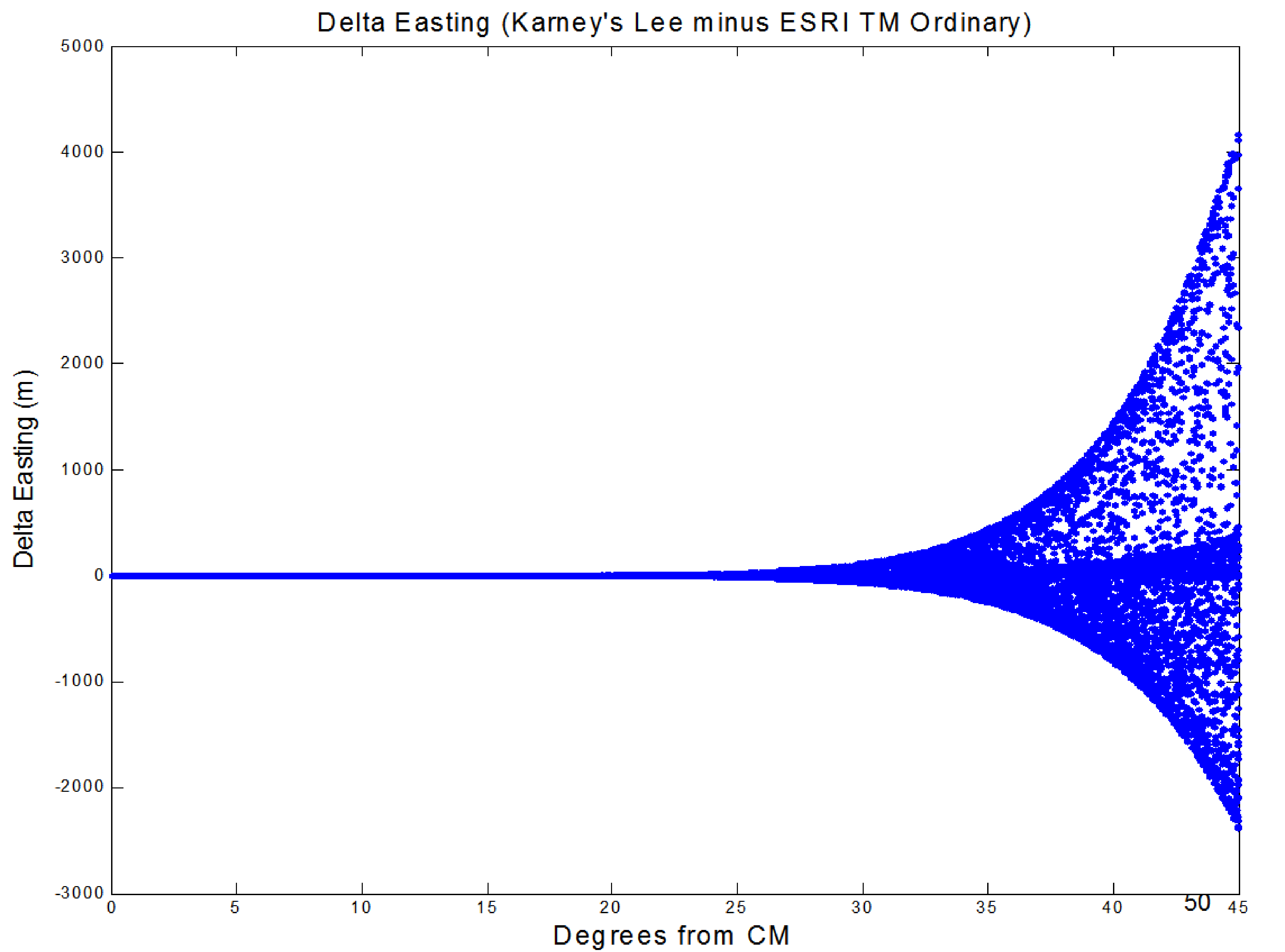
Truth minus the Two ESRI TMs

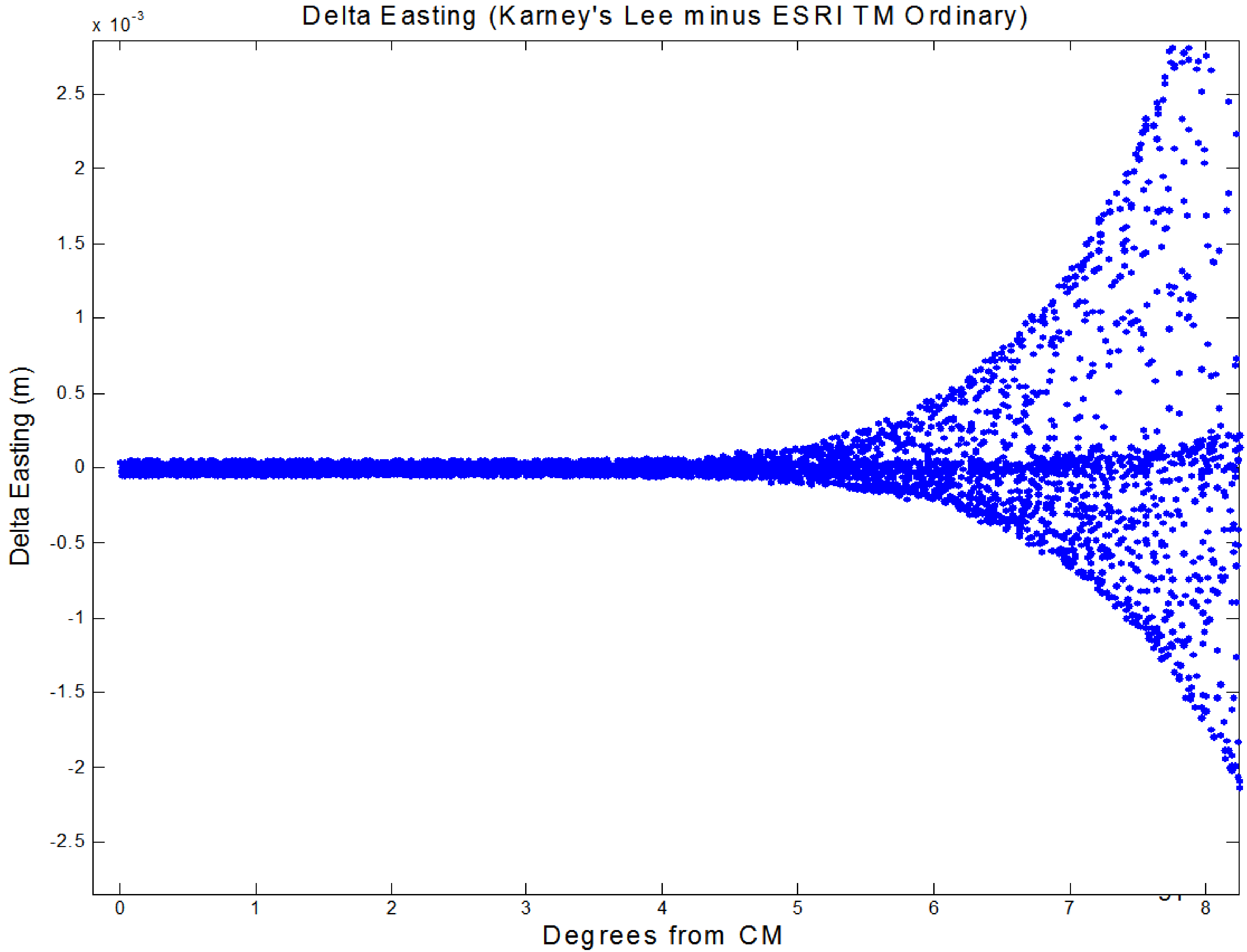
Noel Zinn

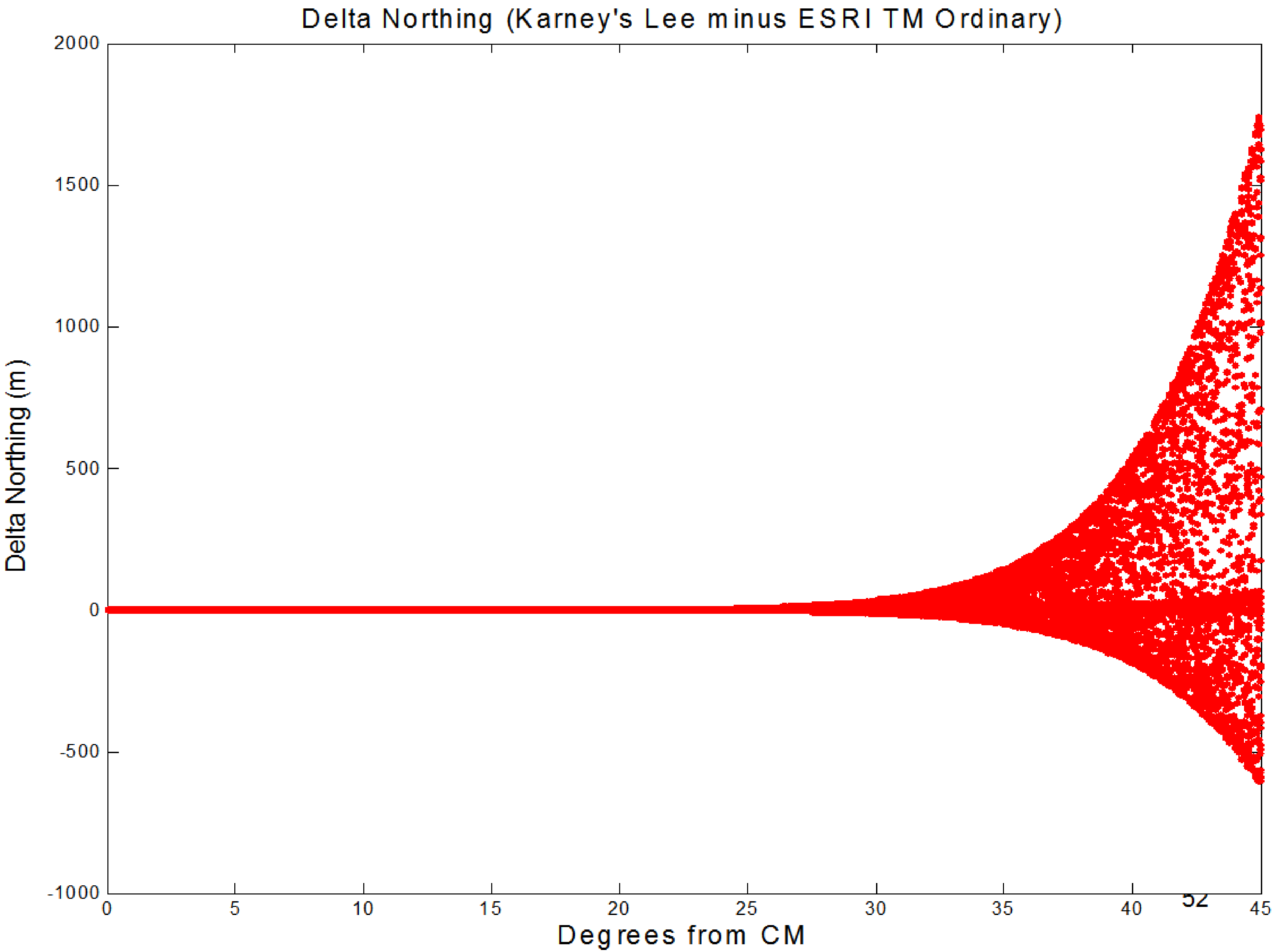
February 12, 2010

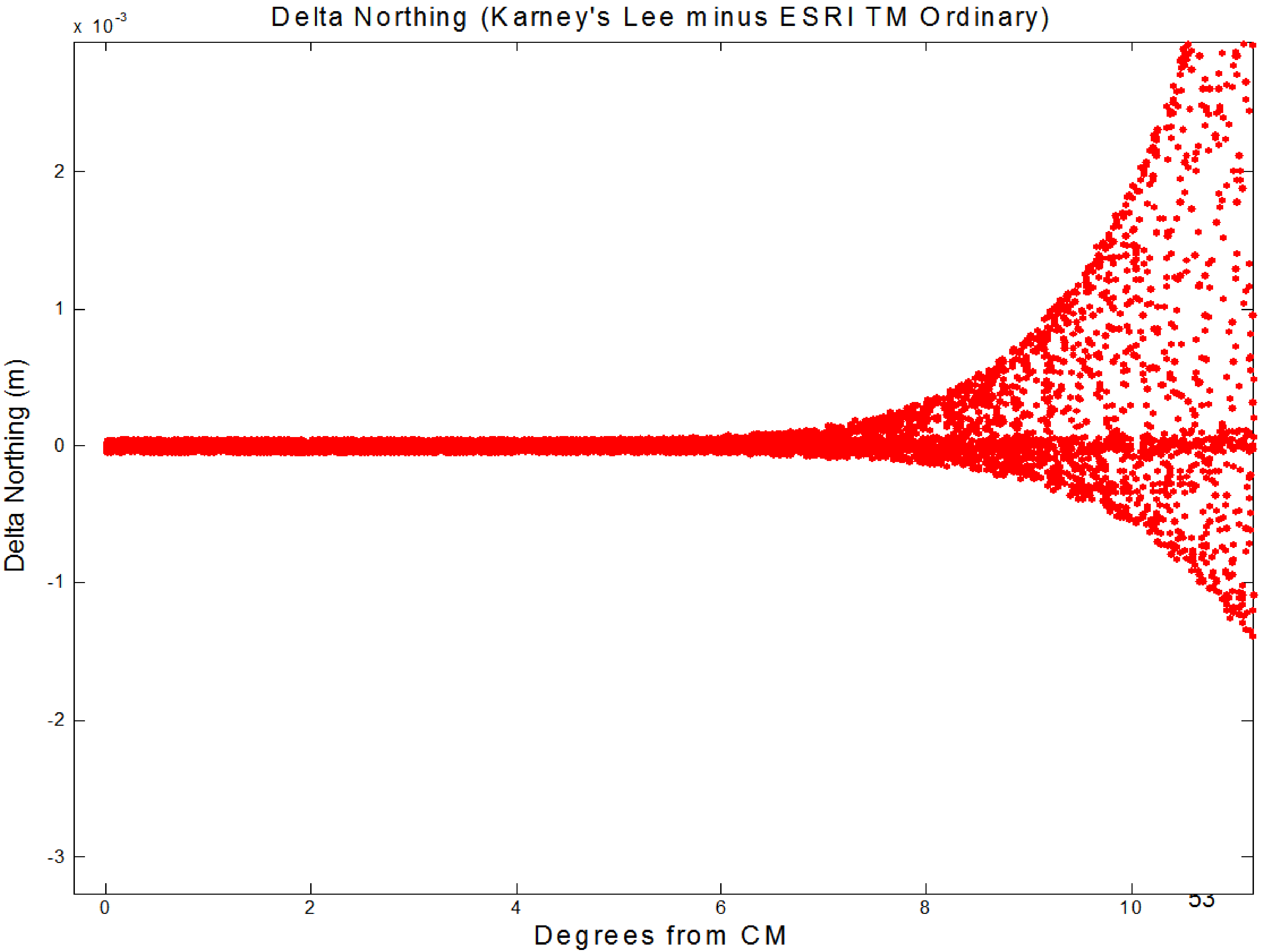
Overview

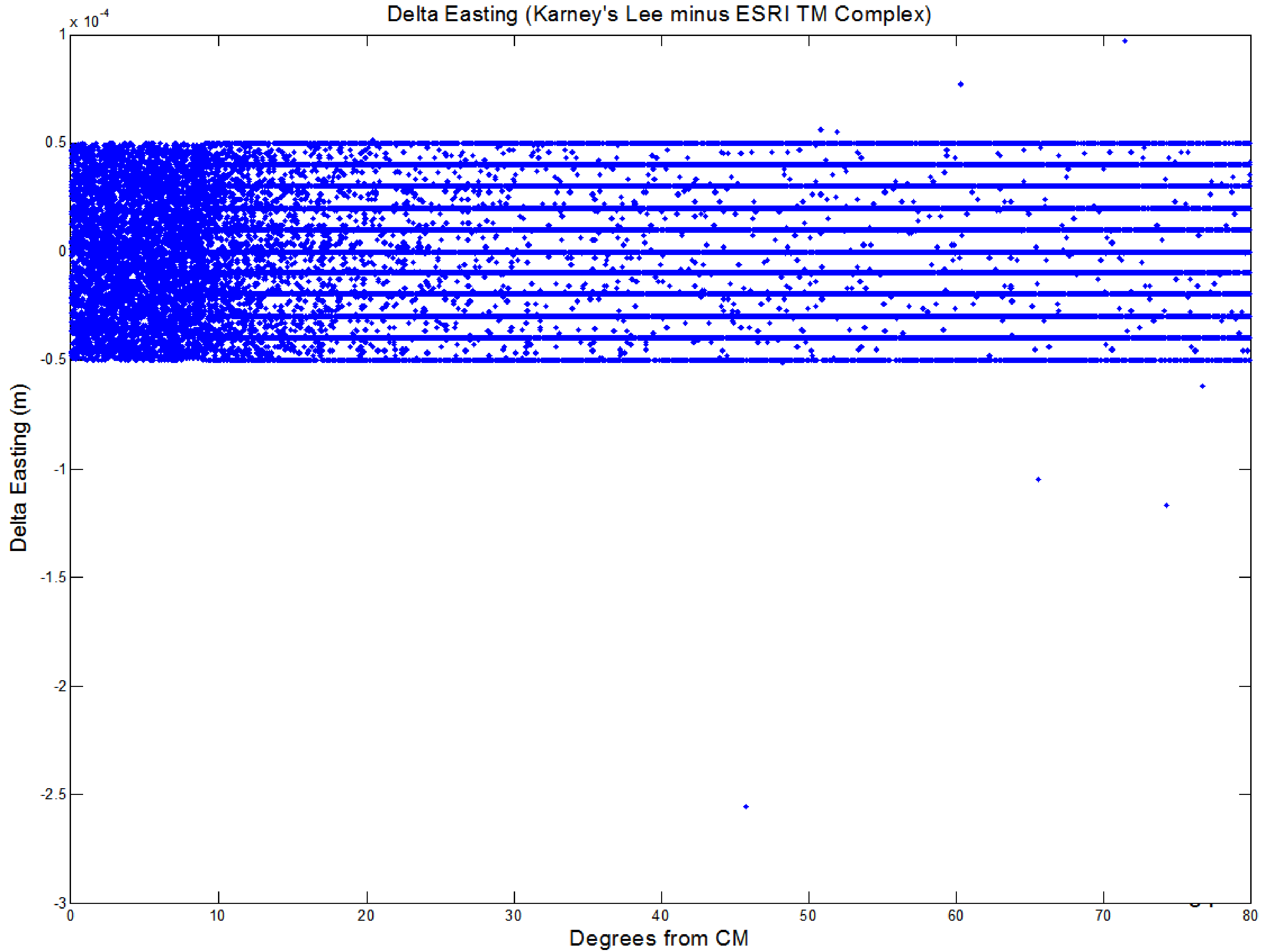
- I have recommended that the Finnish TM (simple algorithmically and good to 40 degrees from the CM) be adopted by OGP (as supported by two previous presentations)
- Here (for the record) are both of the ArcGIS TM projections differenced with Karney's truth points (discussed previously)
- The ordinary TM will compute out to 45 degrees from the CM, but its quality is only a little better than Snyder and a little less than Redfearn
- The complex TM will compute out to 80 degrees from the CM, and its quality is better than a millimeter (the resolution of the numbers presented to me). It could be better.
- See the following comparisons.

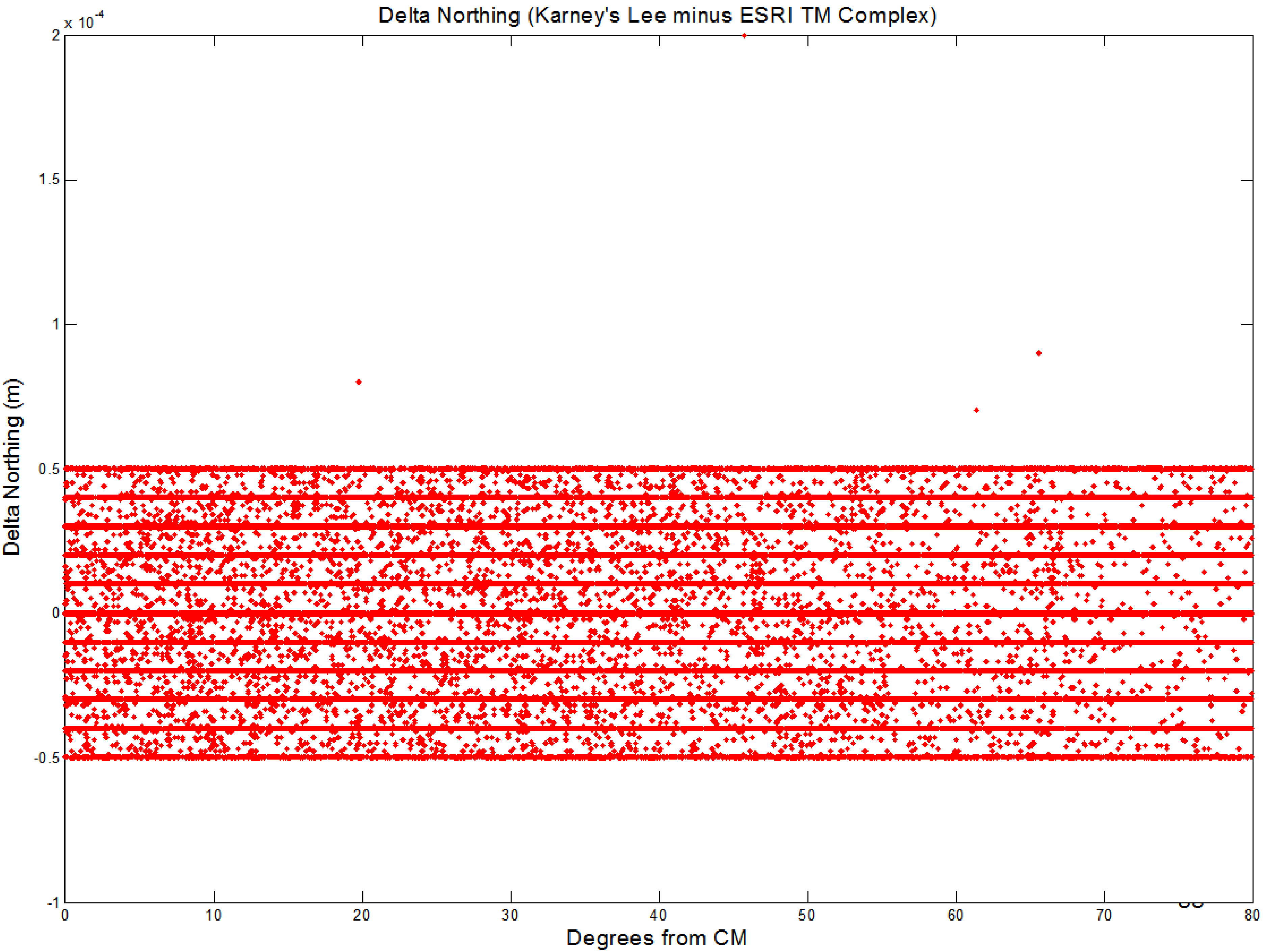

Delta Easting (Karney's Lee minus ESRI TM Ordinary)
Delta Easting (m)
Degrees from CM
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
0
5
10
15
20
25
30
35
40
45


Delta Easting (Karney's Lee minus ESRI TM Ordinary)
x 10^-3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
Delta Easting (m)
0
1
2
3
4
5
6
7
8
Degrees from CM


Delta Northing (Karney's Lee minus ESRI TM Ordinary)
Delta Northing (m)
2000
1500
1000
500
0
-500
-1000
0
5
10
15
20
25
30
35
40
45
Degrees from CM


Delta Northing (Karney's Lee minus ESRI TM Ordinary)
x 10^-3
Delta Northing (m)
2
1
0
-1
-2
-3
0
2
4
6
8
10
Degrees from CM


Delta Easting (Karney's Lee minus ESRI TM Complex)
x 10^-4
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
Delta Easting (m)
0
10
20
30
40
50
60
70
80
Degrees from CM


Delta Northing (Karney's Lee minus ESRI TM Complex)
x 10^-4
2
1.5
1
0.5
0
-0.5
-1
Delta Northing (m)
0
10
20
30
40
50
60
70
80
Degrees from CM